Evolutionary Stability of Keynesian Temporary Equilibrium

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Abstract

A model has been built of a small macroeconomy with demand-constrained firms on the basis of Keynesian investment theory in which the price level plays the "carrier of profits" role. The paper shows that there exists a unique steady-state equilibrium in which the compensation to overhead employees generates the gap between the output price and the average cost including capital cost. There are multiple demand-constrained temporary equilibria near a steady-state equilibrium.Using the evolutionary stability approach, the paper explores the questions of whether and to what extent a capitalist economy is robust. It is found that zero-investment mutant strategy can upset an equilibrium with the smallest population, thus triggering debt-deflation process. Further, the paper demonstrates that, the larger compensation to overhead employees, the more stable the economy becomes. In particular, the steady state equilibrium with no such compensation is evolutionarily unstable.

1 Introduction

In his seminal paper, "The Debt-Deflation Theory of Great Depressions," Irving Fisher argued that, under ordinary conditions, an economy is always

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near stable equilibrium. However, "after departure from it beyond certain limits, instability ensues, [...] such a disaster is somewhat like the 'capsizing' of a ship" (Fisher (1933)). A comprehensive macroeconomic model is needed that can explain and encompass both the fluctuations near the steady state and the process of economic collapse into depression as its special phases.

Currently, dynamic stochastic general equilibrium (DSGE) models (e.g., Gali, Gertler and Lepoz-Salido (2001), Woodford (2003), Christiano et al. (2005), and Smets and Wouters (2007)) are the dominant model building framework in macroeconomics. However, many researchers have questioned the validity of their basic assumptions: a representative agent, rational expectations, and continuous market clearing (Chiarella, Flaschel and Semmler (2012) and Yoshikawa (2012)). As Chiarella et al. (2012) pointed out, the use of these assumptions in the baseline DSGE model implies that an economy exposed to unanticipated shocks moves back to its steady state equilibrium while "but persistent business fluctuations or collapse of an economy remain excluded by the very solution method of rational expectations approac." (ibid, p. xxviii)[1].For this reason, DSGE models are unstuitable as a comprehensive model in the context stated above. In contrast, Hyman Minsky (1919-1996) hypothesized financial instability arises endogenously out of a robust financial system. There have been many studies focused on constructing a comprehensive model on the basis of Minsky's hypothesis (See papers in Semmler (1986).)[2].

This paper first presents a formal model that enables the demand price of productive capital to be expressed in terms of input and output prices as well as an aggregate demand. In Minsky's investment theory, the capitalized value of expected earnings per unit of investment (the demand price of productive capital) is the key variable (Minsky (1986, 2008)), thus has been studied by many researchers (e.g., Taylor and O'Connell (1985), Franke and Semmler (1986)). However, its dependence on depreciation costs has not been a focus. Therefore, the interdependence between output price and aggregate investment has not been clarified. Combining the kinked demand curve perceived by business firms, as posited by Negishi (1979), with Minsky's investment theory clarifies the relationship that enables the output price to assume the "carrier of profits" role (Minsky (1986), p.141). Conversely, by

[1]For, a more detailed explanation, see Kuroki (2013) p.107-108.

[2]For more recent developments in Keynesian macroeconomics, see Chiarella, Flaschel and Semmler (2012, 2013).

incorporating the cost of capital, our model formalizes the insight of Minsky: "the simple equation '*profits equals investment*' is the fundamental relation for a macroeconomics...." (Minsky (1986), p. 144).

Then, the model is used to explore the questions of whether and to what extent a demand-constrained capitalist economy is robust against a sudden change in the reliability of the prospective yield[3]. According to Keynes, "being based on so flimsy a foundation, it (a practical theory of the future) is subject to sudden and violent changes" (Keynes (1937), pp. 214-215). We capture these changes as mutant behaviors and then apply a stability concept similar to an evolutionarily stable outcome (Swinkels (1992)). In the model, depression occurs as Kirman observed: "cycles and fluctuations emerge not as the result of some substantial exogenous shock and the reaction to it of one individual, but as a natural result of interaction, together with occasional small changes or 'mutations' in the behavior of some individuals" (Kirman (1992), p. 133).

An essential assumption of the model is that the investing firms are boundedly rational[4]; they undertake investment projects on the basis of the demand constraints and static expectations for the prices of output and inputs.

Using Fisher's simile of the ship, we find that the mutant strategy of zero investment capsizes an economy at certain limits (i.e., upset its temporary equilibrium by the smallest population of mutants) by precipitating the level of *the output price*, hence expected quasi-rents, so that "the demand price of capital assets is below the supply price; investment in this case will tend to zero" (Minsky (2008), p. 123). This condition in turn triggers a debt-deflation process through negative multiplier effects (*ibid.* p. 125). Since one natural reason for zero investment is liquidation of over-indebtedness, this main result is consistent with the following observation by Fisher (1933): "*the chief interrelations between the nine chief factors may be derived deductively*, assuming to start with, that general economic equilibrium is disturbed by only the one factor of over-indebtedness, and, in particular, assuming that there is no other influence, whether accidental or designed, tending to affect the price level" (Fisher (1933), p. 341).

[3]Keynes made the following observation: "Let us recur to what continuing, much happens at the crisis. [...] The disillusion comes because doubts suddenly arise concerning the reliability of the prospectiveyields, perhaps as the stock of newly produced durable goods steadly increases" (Keynes 1937, p. 317).

[4]The concept of bounded rationality was first introduced by Simon (1947).

Furthermore, three stabilizing factors are identified. The first two, investment and propensity to consume, are intuitively obvious. The more that firms invests, the more stable the economy. Likewise, the higher the propensity to consume, the more stable the economy although at the cost of long-run steady-state performance. The third faqctor, the compensation paid to overhead employees, is not so obvisou and thus has received little attntion from researchers. An increase in the compensation to overhead employees would help stabilize the economy without affecting the long run aggregate output.

Fluctuations in asset prices and the resulting credit and leverage cycles can trigger recession (see, e.g., Kiyotaki and Moore (1997) and Fostel and Geanakoplos (2008), Semmler and Lucas (2012)). Delli Gatti et al. (2005, 2007, 2008) showed that financial fragility and business fluctuations emerge as a result of the complex interactions among firms and the banking system. Delli Gatti et al. (2006, 2009, 2010) demonstrated that the default of one agent can generate a bankruptcy crisis across the network through the productive and credit linkages among firms. These works on leverage cycle and financial fragility focused on financial developments that explain the cause of mutant behaviors; hence, their works and ours are complementary.

The structure of the paper is as follows: Section 2 describes the model. Section 3 shows the existence of a unique steady-state equilibrium and presents its properties. Section 4 defines Keynesian temporary equilibrium and presents factors that affect the range of the equilibria. Section 5 presents stability analyses of Keynesian temporary equilibrium and discusses their implications. Section 6 summarizes the key points and mentions future extensions.

2 Model

This section first presents the sequence of events and overviews the model for introducing a demand-constrained temporary equilibrium. It then describes how a firm facing demand constraint determines employment and investment.

2.1 Description

Time in our model is discrete and is indexed by $t = 0, 1, 2, ...$, with $t = 0$ being the initial time. There are two types of workers: administrative and managerial (hereafter called "overhead employees") and production and technology (hereafter called "workers"). The number of overhead employees and their salaries in real terms are both fixed whereas the number and wage rate for workers change over time. The economy consists of a number of identical firms, households (workers, overhead employees, and shareholders), and a commercial bank. There are three markets: goods, labor, and bonds which have a maturity of one period. Firms produce goods that can be consumed or invested. The only active players in the economy are the firms, which make employment and investment decisions each period on the basis of static expectations. The other players, the households and bank, are passive. The households first decide on consumption (C) on the basis of their income (Y) and accumulated financial wealth (W^H). The bank provides loans to firms by creating demand deposits (balances in checking accounts) and issues one-period bonds and sells them to the household. The interest rates on the lending loans and bonds are the same while the demand deposits yield no interest. Demand deposits serve as the only means of payment and are thus called "money" as well. The money supply is regulated by a monetary authority. Both the interest and wage rates partially adjust toward their equilibrium rates and the output price is perfectly flexible[5]. Until entrants are introduced in Section 5, the firms are identical unless otherwise specified, so the variables for an individual firm and those for the aggregate have the same value.

The sequence of events is as follows:

1. The interest and wage rates are revised and publicly announced; they remain fixed for the duration of the current period.

2. Firms and the bank pay dividends to shareholders. The bank also pays interest to lenders.

3. Firms determine the amount of investment and employment, hence the amount of output. They pay salaries to overhead employees and wages to workers.

4. Each household's consumption spending is determined using a common consumption function.

5. The output price is determined so that the aggregate level of production equals the aggregate level of demand.

[5]The wage rate rigidity in the model of Negishi (1979) is endogenously generated, resulting from kinks in the perceived labor supply function whereas short-run wage rigidity is simply assumed in this model.

6. Aggregate demand is distributed among the firms, thereby determining each firm's revenues and hence profits.

7. The bank issues and redeems bonds.

Next we will explain these steps in more detail.

1. Revise and publicly announce interest and wage rates

By Walras' law, the bond market achieves equilibrium when the money market is in equilibrium. Hence, the interest rate, r_t, is set so that equilibrium in the money market is achieved. Following Friedman (1971), we assume that the desired level of end-of-period real money balances, m_{t-1}^d, is given by

$$m_{t-1}^d = \ell^d(r_t)y_{t-1}, \tag{1}$$

where $\ell^d(\cdot)$ is the inverse of the velocity of circulation[6]. Although it is more realistic to use the real income in period t, y_t, the real income in the previous period, y_{t-1}, is used instead for expositional simplicity. This assumption is reasonable considering that the household estimate y_t on the basis of y_{t-1} when determining the demand for money. Further, for mathematical tractability, it is assumed that $\ell^d(r_t) = \sigma r_t^{-\beta}$, where σ and β are both positive parameters[7]. The equilibrium interest rate r_t balances the money market:

$$\frac{\bar{M}}{P_{t-1}} = m_{t-1}^d = \sigma r_t^{-\beta} y_{t-1}, \tag{2}$$

where $\bar{M}$ is the supply of money regulated by the monetary authority. Thus, r_t depends on lagged real income:

$$r_t = \left(\frac{\sigma Y_{t-1}}{\bar{M}}\right)^{\frac{1}{\beta}}. \tag{3}$$

In the labor market, for give expected price level P_t and full employment level N_f, wage rate W_t is determined in accordance with an augmented Phillips curve so that

$$\frac{W_t - W_{t-1}}{W_{t-1}} = \beta_w\left(\frac{N_{t-1} - N_f}{N_f}\right) + \frac{P_{t-1} - P_{t-2}}{P_{t-1}}, \tag{4}$$

[6] The wealth term is omitted here because subsequent sections focus on the stability of a short-run equilibrium near the steady-state, where capital stock is fairly constant.

[7] This assumption of constant interest-elasticity is widely used in empirical research. See, for example, Goldfeld et al. (1973) and Ericsson (1998).

where β_w is a positive parameter representing the speed of wage adjustment[8].

2. Pay Dividend and interest

Let L_{t-1} denote the end-of-period loan balance of each firm. Each firm pays dividends D_t, at the beginning of period t, to its shareholders. Dividends are required payments[9]. Furthermore, we assume that the firm pays $100r_t$ as a percentage of the dollar value of its capital assets[10]:

$$D_t = r_t(P_{t-1}K_{t-1} - L_{t-1}). \tag{5}$$

Each firm also makes interest payment, r_tL_{t-1} to the bank, out of which r_tB_{t-1} is paid to households. The bank is assumed to incur no operational costs and to pass along all interest income $r_t(L_{t-1} - B_{t-1})$ to the households[11]. The sum of the dividends from the firms and interest from the bank is $r_t(P_{t-1}K_{t-1} - L_{t-1}) + r_t(L_{t-1} - B_{t-1}) = r_t(P_{t-1}K_{t-1} - B_{t-1})$. Therefore, the household's total dividend and interest income is

$$r_t(P_{t-1}K_{t-1} - B_{t-1}) + r_tB_{t-1} = r_tP_{t-1}K_{t-1}. \tag{6}$$

3. Determine investment and employment

Each firm has a linearly homogeneous Cobb-Douglas production function, hence the quantity of production, Q_t, is determined by the number of workers, N_t, and capital stock available at the beginning of period t, K_{t-1}:

$$Q_t = K_{t-1}^{\alpha}N_t^{1-\alpha}, \tag{7}$$

where $\alpha \in (0, 1)$ represents the output elasticity of capital.

[8] Since the unemployment rate can drop below that of natural unemployment, we allow N_{t-1} to exceed N_f.

[9] This view on dividends is shared by Minsky: "Such a firm expects this coming period's gross profits after taxes, and after its required payments on its debts and its dividends to stockholders, ..." (Minsky 2008, p. 105). Chick made the same point. "In Chapters 10 and 11 (of *General Theory*) dividends were treated like interest, because the subject was attraction of these securities to holders" (Chick 1991, p.288).

[10] This assumption is common in agent-based research. See, for example, Galli (2003, 2008). However, there is no concensus among economists about how much of the total payout should be in the form of dividends (see Al-Malkawi et al. (2010))

[11] We could equivalently assume that the government collects the profits of the bank as a tax and transfers them to the households.

Firm		Bank		Household	
$P_{t-1}K_{t-1}$	L_{t-1}	L_{t-1}	M_{t-1}	M_{t-1}	W^H_{t-1}
	E^q_{t-1}		B_{t-1}	B_{t-1}	
				E^q_{t-1}	

Table 1: Consolidated Balance Sheets of Firms, Bank, and Households

Firms obtain investment funds either from retained profits or by borrowing from the bank[12]. The replacement cost, i.e., the price of new investmentgoods (P_{t-1})is used in evaluating the capital stock. Hence, each firm's end-of-period equity value is $E^q_{t-1} = P_{t-1}K_{t-1} - L_{t-1}$. Table 1 displays consolidated balance sheets for firms, the bank, and households.

The total wages to workers is W_tN_t. Firms also pays salaries, $P_{t-1}\bar{H}N_f$, to managers where $\bar{H}$ signifies the fraction of real income of managers per full employment level of employment[13]. Denoting the depreciation rate of capital by δ its capital stock evolves according to

$$K_t = Z_t + (1-\delta)K_{t-1}. \tag{8}$$

Further, the firm's total cost, V_t, is

$$V_t = P_{t-1}\bar{H}N_f + W_tN_t + U_tK_{t-1}, \tag{9}$$

where U_t is the user cost of capital under static expectation, i.e., $U_t = P_{t-1}(r_t + \delta)$[14]. Following Minsky (1986), we treat wage and salary payment to managers as an allocation of profits: "The wage and salary income of those who do not furnish labor required by the techonology embodied in capital assets are viewed as allocation of profits" (Minsky (1986), p. 154). Therefore, salary to managers is not treated as cost here. Define *the average direct cost* a_t as the sum of the workers' wage bill and capital cost per output, so that

$$a_t = \frac{W_tN_t + U_tK_{t-1}}{Q_t}.$$

[12]Equity financing is omitted to avoid the complex problem of determining stock price.

[13]For sake of simplicity, we assume that each firm pays a fixed amount of salaries in real term to managers over entire periods per workforce . We also ignore the effect of the amount of employees on the wage rate of workers by assuming that the number of managers is constant over time.

[14]The real user cost of capital includes capital gain. See, Hall and Jorgenson (1967). The static expectation is reasonable in view of the constant money supply.

The firm makes investment decision based on an estimated cost for investment goods, which is taken as P_{t-1}[15]. We will explain how the firm decides on employment and investment in Section 2.2.

4. Determine household consumption spending

The households of workers, in aggregate, supply a fixed amount of labor, $\bar{N}$. They also determine the demand for consumption goods and money, hence bonds. Let's consider consumption demand first. If we add the total capital income (6) to the total income received by workers and overhead employees, i.e., $P_t\bar{H} + W_tN_t$, the aggregate nominal income of households, Y_t, can be expressed as

$$Y_t = P_{t-1}\bar{H}N_f + W_tN_t + r_tP_{t-1}K_{t-1}. \tag{10}$$

Note that the total financial wealth of households is $\bar{M} + B_{t-1} + E^q_{t-1} = P_{t-1}K_{t-1}$ because there is no outside money in the model. Let C_t denote aggregate consumption spending in real terms. Following Ando and Modigliani (1963), we see that nominal aggregate consumption depends linearly on Y_t and W^H_{t-1}:

$$P_tC_t = c_yY_t + c_w(P_{t-1}K_{t-1}), \quad 0 < c_y < 1,\ 0 < c_w < 1, \tag{11}$$

where c_y and c_w are propensities to consume from the households' flow of income and stock of wealth.

5. Determine output price

The output price is determined as follows. The equilibrium condition in the goods market is given by

$$Q_t = C_t + Z_t. \tag{12}$$

[15] It usually takes multiple periods to complete the building of plant as formulated by Kydland and Prescott (1982). In addition to this time length, a substantial time length is necessary to make plans, decisions, and orders. Christiano and Todd (1996) note: "The other noteworthy feature of investment projects is that they typically begin with a lengthy *planning phase*, during which architectural plans are drawn up, financing is arranged, permits are obtained from various local authorities, and so on." Since the price of the current period investment goods, P_t is available only at the end of each period while investment decision must be made at the beginning of the period, it seems reasonable to assume that, at the beginning of each period, the firm uses the output price in the previous period, P_{t-1} as the estimated price of investment goods.

Since the price level adjusts instantaneously in this market, it is determined by

$$P_t = \frac{c_y Y_t + c_w P_{t-1} K_{t-1}}{Q_t - Z_t}. \tag{13}$$

6. Distribute aggregate demand among firms

Each firm sells Z'_t units of output to other firms at price P_{t-1} as investment goods and C_t units of output at price P_t to consumers, who simply walk into a shop at the end of each period. Hence, the firm's profit is given by[16]

$$\Pi_t = P_t C_t + P_{t-1} Z'_t - V_t. \tag{14}$$

$$= P_t C_t + P_{t-1} Z'_t - \big(P_{t-1} \bar{H} N_f + W_t N_t + U_t K_{t-1}\big). \tag{15}$$

Since the firm can use its internal fund $P_{t-1}\delta K_{t-1}$ to finance the investment, the firm's debt balance evolves according to

$$\begin{aligned} L_t &= L_{t-1} + P_{t-1} Z_t - \{P_t C_t + P_{t-1} Z'_t - (W_t N_t + P_{t-1} \bar{H} N_f + r_t E^q_{t-1} + r_t L_{t-1})\} \\ &= L_{t-1} - \Pi_t + P_{t-1}(Z_t - \delta K_{t-1}), \end{aligned} \tag{16}$$

7. Issue and redeem bonds

Let S_{t-1} be the aggregate balance of liquid assets, i.e., $S_{t-1} = \bar{M} + B_{t-1}$, which is equal to L_{t-1} by Table 1. The supply of bonds outstanding is determined in accordance with (16) to be $B_t = L_t - \bar{M}$. The bank issues new bonds $\Delta B = B_t - B_{t-1}$ so that the new bond balance equals the required bond balance, B_t. From the household' aggregate budget constraint, we have

$$S_t = S_{t-1} + Y_t - P_t C_t. \tag{17}$$

The value of equity, E^q_t, is the residual, i.e.,

$$E^q_t = P_t K_t - L_t.$$

[16] This definition of profit is more like that of retained earnings than the conventionally definition, which treats dividends as a part of profit. Lemma 1 below suggests that our definition is useful for measuring the profitability of investment. Notice that the current payment for investment spending does not appear in V_t. This is because the payment is made form the capital account. For future reference, we define quasi-rent Π^q as revenue minus wages, namely $\Pi^q_t = P_t C_t + P_{t-1} Z'_t - W_t N_t$. We define gross profit, denoted by Π^g, as quasi-rent minus capital cost (including dividends and principal repayment) $\Pi^g_t = \Pi^q_t - r_t(P_{t-1}K_{t-1} - L_{t-1}) - (r_t + \delta)L_{t-1} = \Pi_t + \delta(P_{t-1}K_{t-1} - L_{t-1})$.

Now we are ready to explain the determination of employment and investment.

2.2 Firms in a Demand Deficient Regime

This subsection describes how firms determine the amounts of investment and employment and thus production. This will enable derivation of aggregate employment and investment as functions of prices and aggregate demand, Firms are boundedly rational because they do not engage in any strategic reasoning in forecasting the consequences of their decisions and because they maintain static expectations about prices and aggregate demand. Hereafter, we use lower case letters to deonote individual-level variables and capital letters to denote aggregate variables.

To introduce a new assumption, we index firms by using θ. The labor-capital ratio of firm θ is denoted as $\nu_t(\theta) = n_t(\theta)/k_{t-1}(\theta)$, the investment ratio is denoted as $\zeta_t(\theta) = z_t(\theta)/k_{t-1}(\theta)$, and a strategy pair is represented by $x_t(\theta) = (\nu_t(\theta), \zeta_t(\theta))$. The demand constraint ratios per physical capital for periods t and $t+1$ are respectively represented by $\lambda_t(\theta) = \bar{q}_t(\theta)/k_{t-1}(\theta)$ and $\lambda'_{t+1}(\theta) = \bar{q}_{t+1}(\theta)/k_{t-1}(\theta)$.

To allow for aggregation over firms, we assume that the aggregate demand is allocated to individual firms in proportion to their capital stock, so the $\lambda(\theta)$'s of all firms are identical. We state the proportional demand assumption formally:

Assumption 1. $\forall\theta$, $\lambda_t(\theta) = \lambda_t$, and $\lambda'_{t+1}(\theta) = \lambda'_{t+1}$.

2.2.1 Employment Decision

First, let us describe how a firm determines the level of employment in a demand deficient economy. We follow the methodology for a perceived kinked demand curve developed by Negishi (1979)[17]. According to Negishi, due to imperfect information about prices, asymmetric responses by customers to a price increase versus a price reduction give rise to a kinked demand curve (Figure 1). For the reader's convenience, Appendix A summarizes the results of Negishi (1979) in our context.

[17] The pioneering work by Sweezy (1939) introduced the kinked demand curve in oligopoly theory.

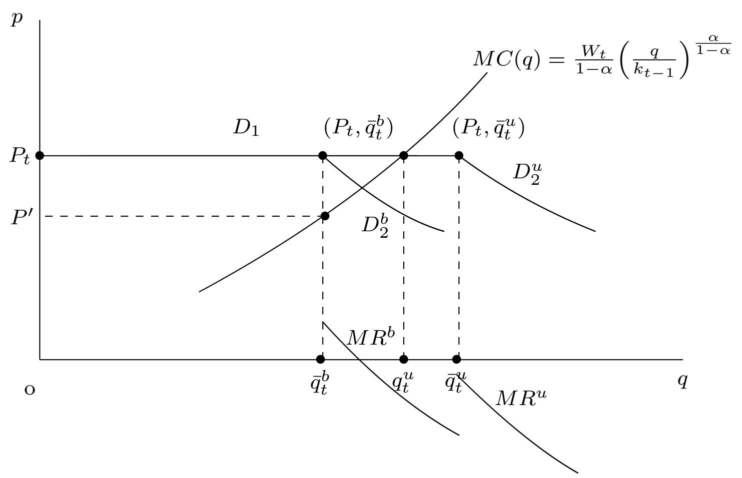

Figure 1: Output Decision using Kinked Demand Curve

Definition 1. A kinked demand curve characterized by $(P_t, \bar{q}_t)$ is the subjective demand schedule that passes through the starting point, $(P_t, \bar{q}_t)$,

$$p = p(q, P_t, \bar{q}_t),$$

such that

$$P_t = p(\bar{q}_t, P_t, \bar{q}_t),$$

and that $p = P_t$ for $q \leq \bar{q}_t$ and $\partial p/\partial q < 0$ for $q > \bar{q}_t$, where q is the possible level of output and p is the price at which an individual firm believes that it can sell quantity q.

For market price P_t and demand constraint $\bar{q}_t$, Negishi (1979) showed that the firm will choose employment level n_t to maximize the current quasi-rent:

$$\pi_t^q = P_t \min(F(n_t), \bar{q}_t) - W_t n_t. \tag{18}$$

In our context, the quasi-rent can be expressed as a function of ν_t and λ_t: $\pi_t^q = \pi^\nu(\nu_t; \lambda_t, k_{t-1}) = k_{t-1}\Big(P_t \min(\nu_t^{1-\alpha}, \lambda_t) - W_t \nu_t\Big)$. The marginal cost is given by

$$MC(W_t, \nu) = \frac{W_t}{F'(n_t)} = \frac{W_t \nu_t^\alpha}{(1-\alpha)}. \tag{19}$$

The optimal labor ratio is given by

$$\nu_t^* = \begin{cases} \left(\bar{q}_t/k_{t-1}\right)^{1/(1-\alpha)} = \lambda^{1/(1-\alpha)} & \text{if } MC_t = W_t\lambda^{\alpha/(1-\alpha)}/(1-\alpha) < P_t \\ \left((1-\alpha)P_t/W_t\right)^{1/\alpha} k_{t-1} & \text{if } W_t \leq P_t < W_t\lambda^{\alpha/(1-\alpha)}/(1-\alpha) \\ 0 & \text{otherwise, i.e., if } P_t \leq W_t. \end{cases} \tag{20}$$

Accordingly, the corresponding short-run output is given by $q_t^* = \min(q_t^u, \bar{q}_t)$, where $q_t^u = ((1-\alpha)P_t/W_t)^{(1-\alpha)/\alpha} k_{t-1}$. In Figure 1, $q_t^* = q_t^u$ when demand constraint is unbinding, i.e., $\bar{q}_t = \bar{q}_t^u$ whereas $q_t^* = \bar{q}^b$ when it is binding, i.e., $\bar{q}_t = \bar{q}_t^b$. This implies that, in the case of an unbinding demand constraint, a firm will increase its employment as long as the marginal cost is below the output price. For future reference we state this observation as a remark.

Remark 1. If $P_t > W_t/F'(n_t) = W_t/\{(1-\alpha)k_{t-1}^\alpha n_t^{-\alpha}\}$ and $F(n_t) = k_{t-1}^\alpha n_t^{1-\alpha} < \bar{q}_t$, the firm can increase profit by increasing the number of workers employed.

In what follows, the notation $\bar{\nu}_t$ denotes the labor-capital ratio under a demand constraint, i.e., $\bar{\nu}_t = (\bar{q}_t/k_{t-1})^{1/(1-\alpha)}$. Also, $\nu_{c,t}$ and $\lambda_{c,t}$ respectively denote the labor-capital ratio and the constraint ratio that minimizes the average direct cost. For a given k_{t-1} and input prices, Figure 2 depicts the marginal cost curve and average direct cost curve, for two different levels of labor-capital ratios $\bar{\nu}^1 < \bar{\nu}$ and $\bar{\nu}^2 > \bar{\nu}$, corresponding to the different levels of the demand constraint. We also define the following relationship between the marginal and average direct costs for future reference:

Remark 2. The marginal cost, $MC(W_t, \bar{\nu}_t)$, is higher (lower) than the minimum of the average direct cost, a_t^*, if $\bar{\nu}_t$ is larger (less) than $\nu_{c,t}$.

2.2.2 Investment Decision

In determining the amount of investment, which is the difference between the desirable and actual amount of capital stock[18], the firms employ the

[18]This formulation is due to Clark (1917). A more general formulation is known as the flexible accelerator model: "capital is adjusted toward its desired level by a constant proportion of the difference between desired and actual capital." (Jorgenson 1971, p. 1111)

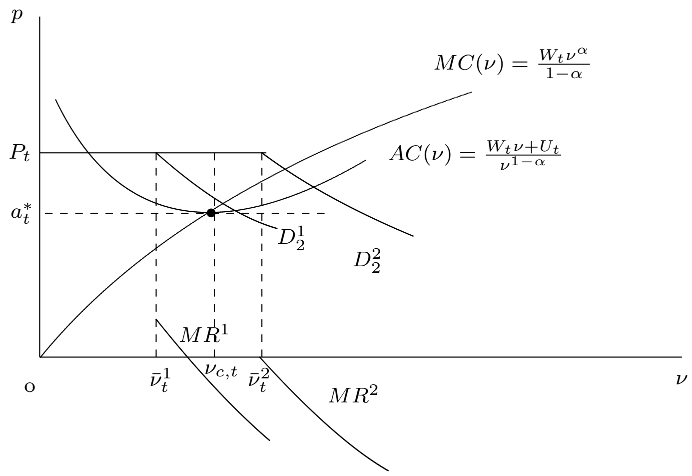

Figure 2: Marginal Cost and Average Direct Cost Curves

investment criterion developed by Minsky (1986, 2008). These boundedly rational firms are assumed to naively judge the profitability of new investment on the basis of static expectations about prices and demand[19].

1. The Demand Price of Capital Stock

Lemma 1 below states that, under static expectations about prices (output price, wage rate, interest rate), the following two conditions are equivalent: (1) the present value of the stream of quasi-rents is greater than the price of investment goods, and (2) output price is greater than the minimum average direct cost. This implies that, provided that prices are expected to be constant over time, the firms undertake new investment only when the output price exceeds the average direct cost, namely $P_t \geq a_t^{*}$[20]. Therefore, if $P_t > a_t^*$, a firm would increase investment as long as the supply capacity is below the expected demand due to the linearly homogenuity of the production function.

A firm facing input prices (W_t, U_t) sets a cost-minimizing labor-capital ratio for production if it can freely choose both capital stock and labor:

$$\min_{n,k}(U_t k + W_t n) \text{ subject to } k^\alpha n^{1-\alpha} \geq 1. \tag{21}$$

The minimum average direct cost of (21) is denoted by a_t^*. The desirable capital-labor ratio, $(k/n)^*$, is derived by the first-order condition for (21):

$$\left(\frac{k}{n}\right)^* = \left(\frac{K}{N}\right)^* = \frac{\alpha W_t}{(1-\alpha)U_t}. \tag{22}$$

Hence, $\nu_{c,t} = (1-\alpha)U_t/(\alpha W_t)$, and $\lambda_{c,t} = \nu_{c,t}^{1-\alpha}$ with $a_t^* = \min_\nu a_t(\nu) = a_t(\nu_{c,t})$. Now, suppose that the firm expects all the prices (including wages and interest rate) to stay constant at the current level, e.g., $P_t = P_{t+1} = \ldots$,

[19] This assumption is justfied by the fact that investment is decision-making under uncertainty. According to Keynes, an entity making a decision under uncertainty assumes that "the present is a much more serviceable guide to the future than a candid examination of past experience would show it to have been hitherto" (Keynes 1937, p. 214).

[20] The equivalence is consistent with the observations made by Minsky: "capital assets yield profits because the output they produce commands a price that exceeds unit out-of-pocket costs. Such a price in excess of the out-of-pocket costs is due to the scarcity of the output and therefore of the capital assets needed to produce the output." (Minsky (1986), p. 179)

$W_t = W_{t+1} = \ldots$, and so on. Under this assumption, let us derive the level of quasi-rents generated by one unit of capital, i.e., $k_t = 1$. For $j = 1, 2, \ldots$, from the cost minimization in (22), the levels of labor input and output are given by

$$n_{t+j} = (1-\delta)^{j-1} \frac{(1-\alpha)U_{t+j}}{\alpha W_{t+j}}, \tag{23}$$

and

$$q_{t+j} = (1-\delta)^{j-1} \left(\frac{(1-\alpha)U_{t+j}}{\alpha W_{t+j}} \right)^{1-\alpha},$$

respectively. Therefore, the wage payment is given by

$$W_{t+j} n_{t+j} = (1-\delta)^{j-1} \frac{(1-\alpha)U_{t+j}}{\alpha}.$$

Let $P^K(P_t, W_t, U_t, \rho)$ denote the capitalized value (with the rate of time preference ρ) of the stream of quasi-rents (from the next period on) that one unit of capital stock is expected to yield. In other words, $P^K(\cdot)$ is the demand price of a capital asset[21]. Then, from the static price expectation, we have

$$\begin{aligned} P^K(P_t, W_t, U_t, \rho) &= \sum_{j=1}^{\infty} \frac{1}{(1+\rho)^j} \pi^q_{t+j}(P_t, W_t, U_t) \\ &= \sum_{j=1}^{\infty} \frac{(1-\delta)^{j-1}}{(1+\rho)^j} \left[P_t \left(\frac{(1-\alpha)U_t}{\alpha W_t} \right)^{1-\alpha} - \frac{(1-\alpha)U_t}{\alpha} \right] \\ &= \frac{1}{\rho+\delta} \left[P_t \left(\frac{(1-\alpha)U_t}{\alpha W_t} \right)^{1-\alpha} - \frac{(1-\alpha)U_t}{\alpha} \right]. \end{aligned} \tag{24}$$

We can interpret $P^K(P_t, W_t, U_t, \rho)$ as the demand price of a productive capital asset. The greater the excess of a price of capital asset over its supply price P_{t-1}, the more attractive a new investment project[22].

The marginal efficiency of capital $\hat{\rho}$ in this context is the rate of time preference that makes $P^K(P_t, W_t, U_t, \rho)$ equal to the supply price of capital

[21] See Minsky (1986) and (2008).

[22] According to Minsky, the value of ρ depends on the interest rate and the virtue of liquidity, which reflects uncertainty felt by entrepreneurs, households, and bankers. See Minsky (1986), Chapter 8, and Minsky (2008), Chapter 5.

P_{t-1}[23]: i.e.,

$$P^K(P_t, W_t, U_t, \hat{\rho}) = P_{t-1}. \tag{25}$$

Lemma 1. Under the static price expectation, the following four conditions of profitability are equivalent: (L1) $P^K(P_t, W_t, U_t, r_t) > P_{t-1}$, (L2) the marginal efficiency of capital exceeds the interest rate, i.e., $\hat{\rho} > r_t$, (L3) the one-period quasi-rent exceeds the user cost of capital, i.e., $\pi_t^q > U_t$[24], and (L4) the output price exceeds the minimum average direct cost, i.e., $P_t > a_t^*$.

Proof. See, Appendix B. □

The next corollary is a direct consequence of Lemma 1. It says that, if the output price equals the minimum direct average cost, a new project will break even .

Corollary 2. $P^K(a_t^*, W_t, U_t, r_t) = P_{t-1}$

Regardless of the level of aggregate demand, if any of the above profitability conditions fails to hold (including break-even projects), the firm will refrain from investment. Otherwise, the level of investment will depend on the demand constraint.

2 Demand Constraint

The level of current investment affects productive capacities in future periods. However, we assume that our bounded rational firms ignore the effects of new investment beyond the next period and thus have an investment horizon of two periods. In other words, in each investment decision, they focus only on the resulting profit in the next period. As the market conditions change, they think that they will have chances to adjust the level of desired capital upon receiving new information in later periods. In other words, they make a series of tentative investment plans.

The firms compute the cost-minimizing capital-labor ratio for period $t+1$ on the basis of W_t and U_t. They calculate k_t^* so that the resulting productive

[23]Keynes defined *the marginal efficiency of capital* as "the rate of discount which would make the present value of the series of annuities given by the returns expected from the capital asset during its life just equal its supply price" (Keynes 1936, p. 135)

[24]Since $\pi_t^q - U_t k_{t-1} = \pi_t^g$, the inequality $\pi_t^q = (P_t q_t - W_t n_t)/k_{t-1} > U_t$ implies that $\pi_t^g = (\pi_t + P_t \bar{h} N_f)/k_{t-1} > 0$. Accordingly, condition (L3) can be expressed as $\pi_t^g > 0$.

capacities just cover the expected demand in period $t+1$, i.e., $\bar{q}_{t+1}$[25]. Provided that $P_t > a_t^*$, this procedure will result in the maximum profits because the production function is linearly homogenous, and it is profitable to produce as much as demand allows. Following an argument similar to the one followed in the previous subsection, we see that it is unprofitable for the firm to build new capital stock ζk_{t-1} beyond the demand constraint. Therefore, if $k_t^* > (1-\delta)k_{t-1}$, $z_t^* + (1-\delta)k_{t-1} \le k_t^*$. Otherwise, $z_t^* = 0$, so that

$$\zeta^* \le \max\left(\eta_t \lambda'_{t+1} - (1-\delta), 0\right),$$

where $\eta_t(W_t, U_t) = \{\alpha W_t / \big((1-\alpha)U_t\big)\}^{1-\alpha}$. Each additional unit of investment yields the present value of quasi-rents, $P^K(\cdot)$, while incurring P_{t-1} as the cost. Therefore, using investment ratio ζ results in the generatation of the expected present value of gross profits, π_t^ζ:

$$\begin{aligned} &\pi_t^\zeta(\zeta_t; \lambda'_{t+1}, k_{t-1}) = P^K(P_t, W_t, U_t, r_t)k_{t-1} \\ &\times \Big[\min\Big\{\max\left(\eta_t(W_t, U_t)\lambda'_{t+1} - (1-\delta), 0\right), \zeta_t\Big\}\Big] - P_{t-1}\zeta_t k_{t-1}. \end{aligned} \tag{26}$$

From equation (22) and the production function, we compute the desired level of capital as

$$k_t^* = \left(\frac{\alpha W_t}{(1-\alpha)U_t}\right)^{1-\alpha} \bar{q}_{t+1}.$$

If no investment is undertaken in period t, the amount of capital that will be available at the beginning of period $t+1$ is $(1-\delta)k_{t-1}$. Hence, the level of profit-maximizing investment is given by

$$\bar{z}_t = \max\Big(\left(\frac{\alpha W_t}{(1-\alpha)U_t}\right)^{1-\alpha} \bar{q}_{t+1} - (1-\delta)k_{t-1}, 0\Big). \tag{27}$$

Thus, the profit maximizing investment ratio, $\bar{\zeta}_t$, is given by

$$\bar{\zeta}_t = \begin{cases} \max\Big(\left(\frac{\alpha W_t}{(1-\alpha)U_t}\right)^{1-\alpha} \lambda'_{t+1} - (1-\delta), 0\Big) & (P_t > a_t^*) \\ 0 & (P_t \le a_t^*). \end{cases} \tag{28}$$

[25] With a new investment z_t, $z_t + (1-\delta)k_{t-1}$ units of capital stock will be available at the beginning of the next period that can produce output $q_{t+1} = (z_t + (1-\delta)k_{t-1})/\{\alpha W_t/\big((1-\alpha)U_t\big)\}^{1-\alpha}$.If this quantity of supply exceeds the demand constraint $\bar{q}_{t+1}$, the firm expects that it will need to cut output price to increase sales. This will give rise to a kinked demand curve for a capital asset. Assuming that the perceived demand curve is kinky enough to induce the firm to build just enough productive capacity to cover the demand constraint, we can follow a procedure similar to the one used for the output decision.

The following lemma, which directly follows from (28), gives the necessary conditions for a positive investment:

Lemma 3. If $\bar{\zeta}_t > 0$, then $P_t > a_t^*$ (hereafter referred to as "AC condition") and $\{\alpha W_t/\big((1-\alpha)U_t\big)\}^{1-\alpha}\lambda'_{t+1} > (1-\delta)$ (hereafter referred to as "demand condition").

The expected total quasi-rents, $\hat{\pi}$, minus cost of investment are given by

$$\begin{aligned}
&\hat{\pi}(\nu_t, \zeta_t; \lambda_t, \lambda'_{t+1}k_{t-1}) = \pi^{\nu}(\nu_t; \lambda_t, k_{t-1}) + \pi^{\zeta}(\zeta_t; \lambda'_{t+1}, k_{t-1}) \\
&\quad = \big\{P_t \min(\nu_t^{1-\alpha}, \lambda_t) - W_t\nu_t\big\}k_{t-1} + P^K(P_t, W_t, U_t, r_t)k_{t-1} \\
&\quad \times \left[\min\Big\{\max\left(\Big(\frac{\alpha W_t}{(1-\alpha)U_t}\Big)^{1-\alpha}\lambda'_{t+1} - (1-\delta), 0\right), \zeta_t\Big\}\right] - P_{t-1}\zeta_t k_{t-1}.
\end{aligned} \tag{29}$$

For given $(P_t, W_t, r_t, U_t, P_{t-1})$, the firm chooses employment and investment ratios in each period to maximize $\hat{\pi}(\nu_t, \zeta_t; \lambda_t, \lambda'_{t+1}, k_{t-1})$. The model is closed if $(\lambda_t, \lambda'_{t+1})$ are specified. A dynamic version of the model is beyond the scope of the current paper (cf. Takahashi and Okada (2013)). We examine a steady-state equilibrium below.

3 Steady-State Equilibrium

This section shows the existence of a unique steady-state equilibrium (SSE) and its properties. In particular, we examine the effects of the propensities to consume, c_y and c_w, and the real compensation to overhead employees, $\bar{H}$, to aggregate levels of output and quasi-rents. The analysis will help understand how these parameters affect the stability of Keynesian temporary equilibrium and the long run performance of the economy.

In a steady-state equilibrium, values of all variables stay constant, e.g., $S_{ss} = S_{t-1} = S_t$, and $L_{t-1} = L_t$, and so on. The following lemma gives a set of six equations that summarizes a steady-state equilibrium.

Lemma 4. $(Q_{ss}, K_{ss}, r_{ss}, Y_{ss}, P_{ss}, W_{ss})$ is a steady-state equilibrium if the following six equations are satisfied:

$$Q_{ss} = K_{ss}^{\alpha}\bar{N}^{1-\alpha} \tag{30a}$$

$$\frac{K_{ss}}{\bar{N}} = \frac{\alpha}{(1-\alpha)(r_{ss}+\delta)}\Big(\frac{W_{ss}}{P_{ss}}\Big) \tag{30b}$$

$$\frac{Y_{ss}}{P_{ss}} = \left(\bar{H} + \frac{W_{ss}}{P_{ss}}\right)\bar{N} + r_{ss}K_{ss} \tag{30c}$$

$$(1-c_y)\frac{Y_{ss}}{p_{ss}} = c_w K_{ss} \tag{30d}$$

$$Q_{ss} = \frac{Y_{ss}}{P_{ss}} + \delta K_{ss} \tag{30e}$$

$$\bar{M} = \sigma r_{ss}^{-\beta} Y_{ss} \tag{30f}$$

Proof. By (4), $N_{ss} = \bar{N}$. Hence equation (30a) follows from the production function. By (17), $Y_{ss} = P_{ss}C_{ss} = c_y Y_{ss} + c_w P_{ss} K_{ss}$, which gives (30d). By equation (8), $Z_{ss} = \delta K_{ss}$. Hence, from (12), we get $Q_{ss} = C_{ss} + \delta K_{ss} = Y_{ss}/P_{ss} + \delta K_{ss}$, hence (30e). Equation (30c) directly follows from equation (10), the definition of Y. Substituting $U_{ss} = P_{ss}(r_{ss}+\delta)$ into (22) yields $K_{ss}/\bar{N} = \alpha W_{ss}/\{(1-\alpha)P_{ss}(r_{ss}+\delta)\}$, hence (30b). Eqaution (30f) directly follows from (2). $\square$

The following result states that there exists a unique steady-state equilibrium under the reasonable parameter restriction.

Proposition 1. Let $\bar{H}' \equiv (c' + \delta - \delta/\alpha)(c' + \delta)^{1/(1-\alpha)}$. If and only if $0 \le \bar{H} < \bar{H}'$, there exists a unique steady-state equilibrium, $(Q_{ss}, K_{ss}, r_{ss}, Y_{ss}, p_{ss}, w_{ss})$ with $r_{ss} > 0$. Moreover, in the steady-state equilibrium, the capital-output ratio is

$$\eta_{ss} = \{c_w/(1-c_y) + \delta\}^{-1} \tag{31}$$

and labor-capital ratio is $\nu_{ss} = \{c_w/(1-c_y) + \delta\}^{1/(1-\alpha)}$.

Proof. See Appendix C. $\square$

In what follows, we will examine the stability of a short run equilibrium near the steady state. An interesting question is whether the steady-state equilibrium itself is stable or not[26]. The following lemma shows that the compensation to overhead employees including managers (compensation to overhead employees) create gap between output price and average indirect cost, which will contribute to enhance stability. Our steady-state equilibrium coincides with the steady-state of descrete-time version of Ramsey-Cass-Koopmans growth model if their consumers have time preference rate that happens to be the steady-state interest rate[27].

Lemma 5. In a steady-state equilibrium, $P_{ss}/a^*_{ss} = 1 - \bar{H}(c' + \delta)^{\alpha/(1-\alpha)}$. Therefore, as $\bar{H}$ increases, the ratio of price level relative to average direct cost, i.e., P_{ss}/a^*_{ss}, rises. In particular, when $\bar{H} = 0$, $P_{ss} = a^*_{ss}$.

Proof. See Appendix D. □

Now we are in a position to ask how the parameters, e.g., $\bar{H}$, c_y and c_w, affect the steady-state variables. The next proposition summarizes the effects of the propensities to consume and the compensation to overhead employees:

Proposition 2. The real steady-state variables, K_{ss}, Q_{ss}, Y_{ss}/P_{ss}, are decreasing in c_y, c_w and independent of $\bar{H}$. The real wage W_{ss}/P_{ss} is increasing in c_y and c_w if $\alpha < 1/2$ and decreasing otherwise. The nominal income and wage rate, Y_{ss}, W_{ss} and price P_{ss} are increasing in c_y and c_w, hence real money supply $\bar{M}/P_{ss}$ is decreasing in them. In contrast, as $\bar{H}$ increases, nominal variables r_{ss}, Y_{ss}, P_{ss}, W_{ss}, as well as real wage W_{ss}/P_{ss} decrease.

Proof. See Appendix E. □

The intuition behind Proposition 2 is as follows: a rise in either propensity (i.e., c_y or c_w) reduces saving, thereby raising the interest rate. This discourages investment, thus decreasing the quantity of productive capital. This in turn reduces output and real wage rate. A higher interest decreases money holding, so that Y_{ss} must expand to keep money demand constant. This implies that P_{ss} should rise. A rise in interest rate has opposing effect

[26] In many models, a steady-state equilibrium is an anchor point. For example, in Ramsey-Cass-Koopmans model, the economy attains equilibrium at the steady-state point by moving along the saddle path toward the point. Once, arrived at the point, there is no further change in consumption or investment.

[27] In our model, the marginal productivity of capital per worker is $\alpha(K/N)^{1-\alpha}$, therefore, the Keynes-Ramsey condition (the Euler equation) in Ramsey-Cass-Koopmans model reduces to, $\alpha(K/N)^{1-\alpha} = \delta + \rho = \delta + r_{ss}$ (See, e.g., Wickens (2008), p. 21.), which is equation (30b) in our model.

on real wage rate. Since labor is a substitute of capital, a rise in r_{ss} pushes up W_{ss}/P_{ss}. On the other hand a reduction in K_{ss} lowers the labor productivity, reducing the real wage. The former effect dominates if $\alpha < 1/2$.

The effects of $\bar{H}$ is a distributional one, and does not affect the real macro variables, such as K_{ss}, Q_{ss} and Y_{ss}/P_{ss}. A rsie in $\bar{H}$ benefits managers but will hurt rentiers, shareholders, and workers.

Further, noting that, by the first-order condition (22), $W_{ss}n_{ss}/(1-\alpha) = W_{ss}n_{ss} + U_{ss}n_{ss}$, in a steady-state, we verify that the marginal cost in the steady-state, denoted by MC_{ss}, is equal to the average direct cost a_{ss}: $MC_{ss} = W_{ss}/F'(n_{ss}) = W_{ss}n_{ss}/\{(1-\alpha)k_{ss}^{\alpha}n_{ss}^{1-\alpha}\} = a_{ss}^*$. If the economy in period t is in a steady-state, the output price P' in Figure 1 equals a_{ss}^*. For $\bar{H} > 0$, from lemma 5, the steady-state output price P_{ss} is strictly greater than a_{ss}^* so that the firms are demand constrained even in the steady-state, which we state as a remark.

Remark 3. If $\bar{H}$ is positive, the firm faces demand shortage even in the steady-state, namely $P_{ss} > MC(q_{ss}) = a_{ss}$.

To examine the stability of a short run equilibrium near the steady-state equilibrium, Keynesian temporary equilibrium is formally defined.

4 Keynesian Temporary Equilibrium

This section defines Keynesian temporary equilibrium (KTE) and examines conditions for which a KTE exists under the assumption that the economy stayed in a steady-state equilibrium up to period $t-1$. Then, by examining the relationship between the compensation to overhead employees and the range of KTE, we show that if the compensation is zero, no KTE with unemployment exists, and that the range of KTE expands as the the compensation increases.

4.1 Definition of Keynesian Temporary Equilibrium

Expecting the current price level to be P_t and the aggregate effective demand to be Q_t at the beginning of period, the firms are choosing the levels of investment and employment, hence also output, under the constraints on their demand. These decisions collectively determine the aggregate level of income, thus also consumption, generating aggregate demand. Roughly speaking, in a Keynesian temporary equilibrium, the resultant aggregate demand is consistent with the initial expectation about the aggregate demand, i.e., Q_t[28]. Our equilibrium concept is similar to "Keynes-Negishi equilibrium" (Negishi (1979), Drèze and Herings (2008))[29]. The equilibrium concept also requires that the money wage rate and interest rate are consistent with their market conditions. In this section, we continue to assume that all the firms are identical. Let Ω_t denote a set of the predetermined macroeconomic variables, i.e.,

$$\Omega_t = (K_{t-1}, W_t, r_t, Y_{t-1}, P_{t-1}, P_{t-2}, W_{t-1}, N_{t-1}).$$

We define a Keynesian temporary equilibrium as follows:

Definition 2. A Keynesian temporary equilibrium is defined by $(P_t, Q_t, \Omega_t, \bar{x}_t)$, where P_t, Q_t, Ω_t, and x_t are the expected price level, the expected aggregate demand, a set of predetermined macro-variables, and a pair of strategies $\bar{x} = (\bar{\nu}_t, \bar{\zeta}_t)$ such that:

1. Facing demand constraints, $\lambda_t = \lambda'_{t+1} = Q_t/K_{t-1}$, each firm chooses $\bar{\nu}_t$ and $\bar{\zeta}_t$ that maximize $\hat{\pi}_t$ in eq. (29) so that equations (20) and (28) are satisfied. The resultant aggregate employment and investment are $N_t = \bar{\nu}_t K_{t-1}$ and $Z_t = \bar{\zeta}_t K_{t-1}$ respectively.

2. The output price is higher than the marginal cost, i.e., $P_t > MC(W_t, \bar{\nu}_t)$.

3. The resultant investment is strictly positive so that, from lemma 3,

$$P_t > a_t^* = \left(\frac{U_t}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \quad \text{and} \quad \left(\frac{\alpha W_t}{(1-\alpha)U_t}\right)^{1-\alpha} \lambda'_{t+1} > (1-\delta).$$

4. The aggregate demand equals the aggregate supply:

$$Q_t = K_{t-1}^{\alpha} N_t^{1-\alpha} = C_t + Z_t, \tag{32}$$

[28]This property is innocuous in the short-run. Keynes notes, "... the theory of effective demand is substantially the same if we assume that short-period expectations are always fulfilled" (Keynes (1973), p. 181)

[29]The rigidity of wage rate is endogenously resulted from kinks in perceived labor supply function in Negishi (1979) whereas our model simply assumes contemporaneous wage rigidity. In Negishi (1979), investment is exogenously given whereas it is endogenous in our model. Drèze and Herings (2008) proved the existence of "Keynes-Negishi equilibrium" in general equilibrium setting. The terminology "Keynes-Negishi equilibrium" was coined by them.

which is also expressed as

$$P_t(Q_t - Z_t) = c_y Y_t + c_w P_{t-1} K_{t-1}, \tag{13'}$$

where $Y_t = P_{t-1}\bar{H}\bar{N} + W_t N_t + r_t P_{t-1} K_{t-1}$.

5. The money market is in equilibrium so that the interest is set to satisfy eq. (3), i.e., $r_t = (\sigma Y_{t-1}/\bar{M})^{1/\beta}$.

6. The nominal wage rate, W_t, is set so that

$$\frac{W_t - W_{t-1}}{W_{t-1}} = \beta_w \Big(\frac{N_{t-1} - \bar{N}}{\bar{N}}\Big) + \frac{P_{t-1} - P_{t-2}}{P_{t-1}}.$$

Some comments are in order. First, for expositional convenience, we require $\lambda_t = \lambda'_{t+1}$, that is, the firms expect the demand for the current period and the long-run expected demand for investment decision are the same. This assumption seems reasonable if one period is short. Second, the condition that $P_t > MC_t$ (hereafter called MC condition) implies that the demand constrain λ_t is actually binding. Third, the profitability condition $P_t > a_t^*$ in condition 3 implies that output price is higher than the shut-down price, i.e., $P_t > W_t$, Therefore, $\bar{\nu}_t > 0$ from eq. (20). Consequently, with the assumption of static expectations about prices, each firm makes investment and employment decisions on the basis of the perceived demand function, characterized by (P_t, λ_t). Finally, it follows from the first condition of KTE, $\lambda'_{t+1} = Q_t/K_{t-1}$, that $\lambda'_{t+1} = \bar{\nu}^{1-\alpha}$. Denoting $\eta_t = (\alpha W_t)/\{(1-\alpha)U_t\}^{1-\alpha}$, the 'demand condition' can be expressed as $\eta_t \bar{\nu}^{1-\alpha} > 1 - \delta$.

4.2 The Existence of KTE

We examine the existence of KTE in period t assuming that the economy remained in the steady-state equilibrium up to period $t-1$. In addition to mathematical tractability and expositional simplicity, we think this assumption to be reasonable for the following reasons. First, if there were no KTE near the steady-state equilibrium, it is unlikely for any KTE to exist. Second, if a steady-state equilibrium is stable in our model as it is the case in Ramsey-Cass-Koopmans growth model, the economy is likely to remain near the SSE once it is achieved. One typical such situation is a KTE that arises when an economy departs from the SSE one period before. Using Fisher's simile of a ship quoted earlier, we treat an original stable position as a SSE, and a ship, which is tipped by some force from the original position, as an economy in a KTE. Before asking below to what extent the ship is stable, we will examine the existence and the properties of KTE.

Let ν_δ satisfy the level of ν that satisfy the 'demand condition' with equality, i.e., $\eta_{ss}\nu_\delta^{1-\alpha} = 1-\delta$. (Thus, KTE exists only when $\bar{\nu} \geq \nu_\delta$.) The following lemma, which states that the output price increases more rapidly than the marginal cost as aggregate demand increases, is necessary for stability analysis.

Lemma 6. Assume that an economy had stayed in a steady-state equilibrium until period $t-1$. If $c_w < (1-\alpha)(1-c_y)$ and $\alpha < c_y$, then $dP(\bar{\nu})/d\bar{\nu} > dMC(W_t, \bar{\nu})/d\bar{\nu}$ for $\bar{\nu} \in (\nu_\delta, \nu_1)$.

Proof. See Appendix F. □

To prepare for proving the existence of KTE, let the labor endowment be denoted by N_{max} ($> \bar{N}$), and the maximum attainable labor-capital ratio by $\nu_1 = N_{max}/K_{ss}$. Further, it is shown below that, there is a threshold value of $\bar{H}$, denoted by $\bar{H}_\delta$, such that for $\bar{H} \in [0, \bar{H}_\delta)$, ν' can be defined as the level of ν that satisfies the 'AC condition' with equality, i.e., $P_t(\nu') = a^*_{ss}$, and that such a ν' exists between ν_δ and ν_{ss}. Given that $\bar{H}$ is less than $\bar{H}_\delta$, if the aggregate demand is small enough to satisfy $\bar{\nu} \in (\nu_\delta, \nu')$, the firms do not undertake new investment because the 'AC condition" fails to hold although the "demand condition" is satisified. Moreover, the aggregate demand needed to raise the price level above the average direct cost decreases as the compensation to overhead employees increases.

Lemma 7. If $\bar{H} = \bar{H}_\delta$, $\nu' = \nu_\delta$. For $\bar{H} < \bar{H}_\delta$, the value of ν' is greater than ν_δ and increases as $\bar{H}$ decreases. If $\bar{H} > \bar{H}_\delta$, for all $\bar{\nu} > \nu_\delta$, $P(\bar{\nu}) > a^*_{ss}$.

Proof. See Appendix G. □

Denote $\nu_0 = \max(\nu_\delta, \nu')$. The following proposition shows that there are multiple Keynesian temporary equilibriua around SSE if $\bar{H} > 0$. (Figure 3)

Proposition 3. For each $\bar{\nu}_t \in (\nu_0, \nu_1]$, there exists a corresponding KTE, denoted by $\big(P_t(\bar{\nu}_t), \bar{\nu}_t^{1-\alpha}K_{ss}, \Omega_t, (\bar{\nu}_t, \bar{\zeta}_t)\big)$, where $\bar{\zeta}_t(\bar{\nu}_t) = \eta_{ss}\bar{\nu}_t^{1-\alpha} - (1-\delta)$ and

$$P_t(\bar{\nu}_t) = \frac{c_y W_t \bar{\nu}_t + E_{ss}}{\bar{\nu}_t^{1-\alpha} - \bar{\zeta}_t}, \tag{33}$$

where $E_{ss} = P_{ss}C_{ss}/K_{ss} - c_y W_{ss}\nu_{ss}$.

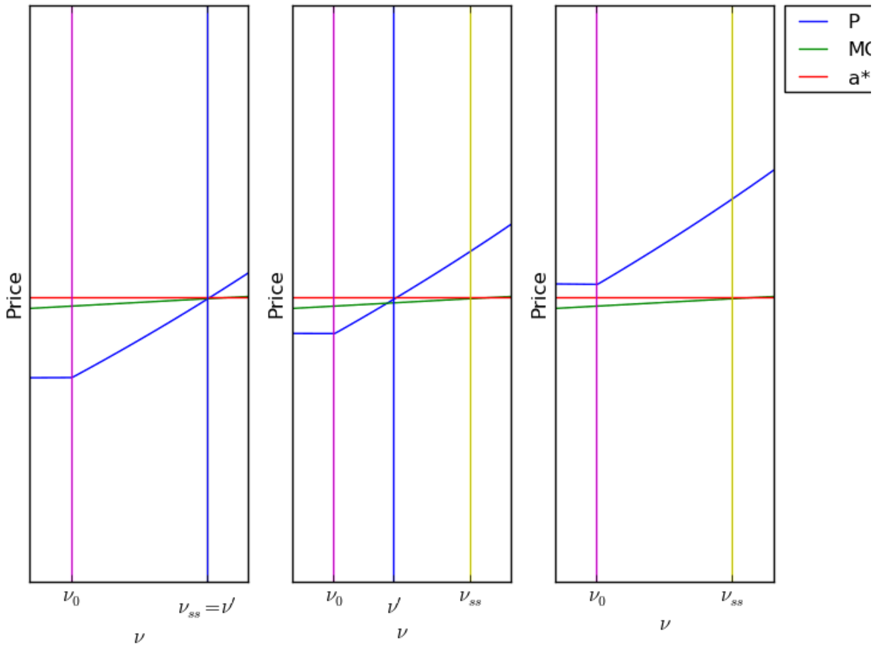

Figure 3: Output Price Curves for Different Values of $\bar{H}$

Proof. Let $\lambda_t = \lambda'_{t+1} = \bar{\nu}_t^{1-\alpha}$. Suppose that $\bar{\nu}_t \in (\nu_0, \nu_{ss})$, then by definition of ν' and remark 2, Lemma 7 implies that $MC_t(W_t, \bar{\nu}_t) < a_t^* < P_t(\bar{\nu}_t)$. By lemma 5, $P_t(\nu_{ss}) = P_{ss} \geq a_{ss}^* \; \forall \; \bar{H} \geq 0$. Then, by lemma 6 and remark 2, for $\bar{\nu}_t \in [\nu_{ss}, \nu_1]$, $P_t(\bar{\nu}_t) > MC(W_t, \bar{\nu}_t) \geq a_{ss}^*$. Thus, $\bar{\nu}_t$ satisfies (20). Moreover, $\eta_{ss}\bar{\nu}_t^{1-\alpha} > 1 - \delta$ since $\bar{\nu}_t > \nu_\delta$. Therefore, $\bar{\zeta}(\bar{\nu}_t)$ satisfies (28). By (33), goods market equilibrium (32) is achieved at $\bar{\nu}_t$. Since $Y_{t-1} = Y_{ss}$, $r_t = r_{ss}$. Moreover, $W_t = W_{ss}$ since $N_{t-1} = N_{ss} = N_f$, and $P_{t-2} = P_{t-1} = P_{ss}$. Consequently, conditions 5 and 6 are fulfilled. □

Corollary 8. Assume that an economy with $\bar{H} = 0$ has stayed in a steady-state up to period $t - 1$. Then there is no Keynesian temporary equilibrium with unemployment.

Proof. By Lemma 5, if $\bar{H} = 0$, $P_{ss} = a_{ss}$. If there were a KTE with unemployment, it must be the case that $N_t < N_{ss} = \bar{N}$, hence $\bar{\nu}_t < \nu_{ss}$. This implies, by lemma 6, that $P_t < a_t^* = a_{ss}$, thus violating the profitability requirement, $P_t > a_t^*$. □

Proposition 3 together with lemma 7 implies that the range of KTE equilibria expands as $\bar{H}$ increases.

5 Stability of Keynesian Tmporary Equilibrium

Now we are ready to ask how stable a Keynesian temporary equilibrium is, and when the equilibrium is upset what will be the consequence. In this section, we first define the degree of robustness of a KTE using the stability concept developed by Swinkels (1992). Then, we show that the equilibrium is most easily upset to precipitate the economy into debt-deflation when a certain population of firms stop investing. Finally, we investigate what factors and to what extent these factors influence the degree of stability. We will show that, the more actively firms undertake investment and the larger the compensation to overhead employees, the more robust the economy becomes. In particular, if the compensation is zero, any steady-state equilibrium is unstable. Throughout this section, the output price, which reflects the scarcity of capital goods, plays an essential role.

5.1 Mutants and Spillover Effect

Supose that an economy in period t is in a Keynesian temporary equilibrium in which incumbent firms play a pair of strategy, $\bar{x} = (\bar{\nu}, \bar{\zeta})$. Throughout the section, we ommit time subscript t whenever obvious. We treat a firm playing $\hat{x} = (\hat{\nu}, \hat{\zeta}) \neq (\bar{\nu}, \bar{\zeta})$ as a mutant[30]. Imagine that, all of a sudden, facing a need to liquidate debt, or to restore health in balance sheet, a certain fraction of firms reduce investment. Or, out of some exogenous shocks, e.g., foreign-originated financial contraction or natural disaster, just imagine some fraction of firms become pessimistic about the future, thus refraining from investment.

In defining the robustness of a KTE, we use stability concept similar to Swinkels's (1992) equilibrium evolutionarily stable (EES) sets[31]. Let θ (index for firms) be distributed uniformly between $[0, 1]$. We say a group of of population size ϵ with mutant strategy $\hat{x} = (\hat{\nu}, \hat{\zeta})$ enters the economy of incumbent strategy pair $\bar{x} = (\bar{\nu}_t, \bar{\zeta}_t)$ in period t if the following two sets of conditions hold:

(1) $\forall \theta \in [0, \epsilon]$, $x_t(\theta) = \hat{x}$, and

(2) $\forall \theta \in (\epsilon, 1]$, $x_t(\theta) = \bar{x}$.

Let $\hat{N} = \hat{\nu} K_{t-1}$ and $\hat{Z} = \hat{\zeta} K_{t-1}$ denote the total employment and investment under an assumption that all players become entrants. We denote the post-entry value of X by $\tilde{X}$. Then, the entry of mutants cause changes in the aggregate employment, output, and investment so that $\tilde{N}(\epsilon, \hat{\nu}) = (1-\epsilon)N_t + \epsilon\hat{N}$, $\tilde{Q}(\epsilon, \hat{\nu}) = K_{t-1}^{\alpha}\{(1-\epsilon)N_t^{1-\alpha} + \epsilon\hat{N}^{1-\alpha}\}$ and $\tilde{Z}(\epsilon, \hat{\zeta}) = (1-\epsilon)Z_t + \epsilon\hat{Z}$, respectively. The employment strategy of mutants change both the aggregate demand and supply. In contrast, their investment will affect only the aggregate demand due to the one-period time lag required to build productive capital. It is convenient to define a post-entry environment:

Definition 3. Define $\mathcal{E}(\epsilon, \hat{x})$ as an economy that arises after a group of mutants of a population ϵ with the strategy $\hat{x}$ enter an economy in a Keynesian temporary equilibrium, in which the incumbents play a strategy $\bar{x}$.

[30]Introducing mutants will make the stability analysis of an equilibrium more compelling whereas the representative agent models cannot as Kirman (1992) stated: "it is contradictory to begin with a single representative agent and then to envisage different individual actions which lead the economy back to equilibrium" (p. 121).

[31]In Swinkels (1992), each player has finite pure strategy set and is allowed to play mixed strategy. Here, since their strategy sets are infinite, either a mutant or incumbent plays a pure strategy.

We also denote the set of best responses to the post-entry environment by $BR(\mathcal{E}(\epsilon, \hat{x}))$. Mutants are endowed with the restricted ability to select a strategy which best responds to the post-entry environment. We require robustness only against such mutants[32]. The mutant strategy $\hat{x}$ is called *equilibrium entrants* if it is a best reply to $\mathcal{E}(\epsilon, \hat{x})$ (Swinkels, 1992).

Regarding the demand constraint, by Assumption 1, a change in aggregate demand, which is equal to the change in output, affects all firms across the economy in proportion to their capital stock, i.e., $k_{t-1}(\theta)$. Therefore, each firm, whether it may be a mutant or an incumbent, perceives its demand constraint ratio to be $\tilde{\lambda}(\epsilon, \hat{\nu}) = (1-\epsilon)\bar{\nu}_t^{1-\alpha} + \epsilon\hat{\nu}^{1-\alpha}$. For instance, if the aggregate demand increases, then all the firms can enjoy its proportional share[33]. The following remark is a consequence of this assumption.

Remark 4. For $\forall \epsilon \in (0,1)$, if $\hat{\nu} < \bar{\nu}_t$, then $\hat{\nu}^{1-\alpha} < \tilde{\lambda}(\epsilon, \hat{\nu})$. Conversely, if $\hat{\nu} > \bar{\nu}_t$, then $\hat{\nu}^{1-\alpha} > \tilde{\lambda}(\epsilon, \hat{\nu})$.

A similar argument applies to a mutant who sets a lower investment ratio than that of incumbents, i.e., $\hat{\zeta} < \bar{\zeta}_t$. Intuitively, the aggregate future demand, that is equal to the aggregate productive capacity by the free movement of price, will be proportionally distributed among producers. Hence, some part of demand foregone by mutants will spill over to other producers. We set out this observation as another remark:

Remark 5. If $\hat{\zeta} < \bar{\zeta}_t$, then the mutants expect unbinding demand constraint in period $t+1$. (For more detail, see Appendix H.)

5.2 Stability of Keynesian Temporary Equilibrium

There are various strategy options that are available to mutants. Some mutants may employ small amount of labor but undertake a large volume of investment. Others may undertake no investment. To each combination of strategy pairs played by incumbents as well as mutants corresponds a minimum required population of mutants, ϵ, that can upset the KTE. In what

[32]The idea is that mutants playing a strategy which is not an optimal one will disappear, being replaced by those who are best responding to post-entry environment. Notice though, unlike in Swinkels (1992), our agents are endowed with the limited capacity and foresight.

[33]This assumption is consistent with the observation made by Tobin: "In demand constrained regimes, any agent's increase in demand—for example, more investment spending by a business firm—has positive externalities." (Tobin (1993), p. 50)

follows, we seek a pair of mutants' strategy $(\hat{\nu}, \hat{\zeta}) \neq (\bar{\nu}, \bar{\zeta})$ that enable the mutants to invade successfully with the smallest population. In other words, the KTE is stable if mutants come in less than this population share. To make the meaning of the stability precise, we need to define *robustness* of a KTE as follows.

Definition 4. A Keynesian temporary equilibrium is said to be robust with a degree of $\hat{\epsilon}$ against equilibrium entrants if there exists $\hat{x}'$ such that $\hat{x}' \in BR(\mathcal{E}(\hat{\epsilon}, \hat{x}')$ and condition (34) below holds for all $\hat{x} \neq \bar{x}$ and $\epsilon \in (0, \hat{\epsilon})$

$$\hat{x} \notin BR(\mathcal{E}(\epsilon, \hat{x})). \tag{34}$$

We call $\hat{\epsilon}$ an invasion barrier (see Chapter 2 of Weibull (1995), for a detailed discussion) and $\hat{x}'$ the most destructive strategy.

1. Intuitive explanation of main results

At individual level, a unilateral *increase* in employment by a mutant should not be profitable because most of the new demand it has created will spill over to other firms. Similarly, a unilateral reduction in profitable employment should be also harmful because it would forego some of the current demand due to the insufficient production capacity. The same argument applies to the investment decision; a unilateral *increase* in investment spending will make a mutant worse off due to the spill-over effect while a unilateral reduction in profitable investment will make it worse off by foregoing some of future demand due to the insufficiency of production capacity. Intuitively, a decrease in investment spending is justifiable only when the resulting reduced price makes new investment no longer profitable. On the other hand, if the mutant producess as much as other firms, the current period aggregate demand remains the same because price falls promptly to expand consumption demand to cancell out the reduced investment spending by mutants until the aggregate demand becomes equal to the current level of supply. These considerations suggest that the most destructive mutant type is $(\hat{\nu}, \hat{\zeta}) = (\bar{\nu}_t, 0)$. The next question is how big their population should be to upset an equilibrium. The larger the level of incumbents' investment, the higher the output price becomes. Therefore, a larger size of mutants is required to reduce the price low enough to stop investing in new projects. Lemma 9 formally states these intuitions.

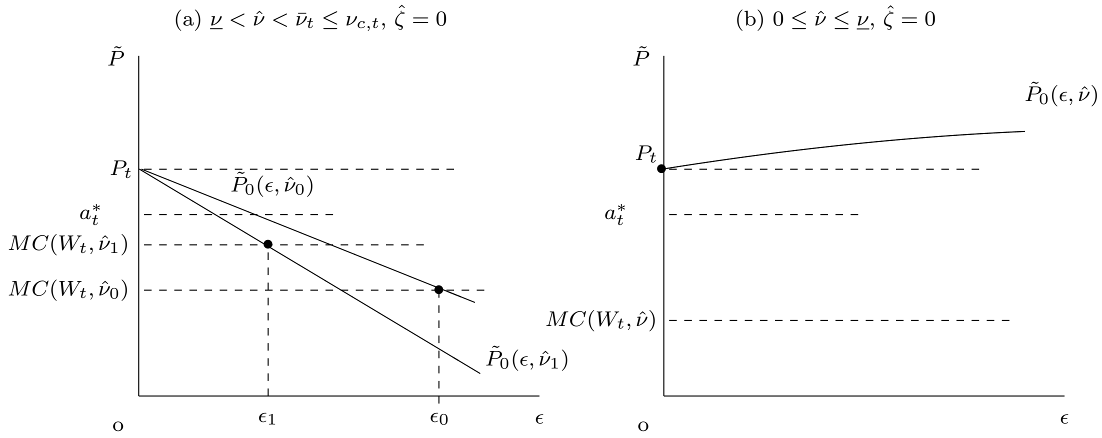

Figure 4: Choice of $\hat{\nu}$

The entry of mutants will affect the aggregate supply and demand, thus causing the post-entry output price to change. Intuitively speaking, in what follows we demonstrate that the equilibrium entrants that upset the KTE most effectively should be able to bring the post-entry price below the direct average cost so that new investment becomes no longer profitable. Those mutants can efficiently reduce aggregate excess demand by producing as much as incumbents but stopping investment.

Figure 4 depicts, for given $\hat{\zeta} = 0$, the equilibrium output price $\tilde{P}_0(\epsilon, \hat{\nu})$ as a function of $\hat{\nu}$ and ϵ. A mutant stops investing when output price falls below a_t^* (the minimum direct average cost) whereas $\hat{\nu}$ becomes the best responding employment ratio if the postenrty proce equals the marginal cost $MC(W_t, \hat{\nu})$. Figure 4-(a) illustrates the effect of $\hat{\nu}$ on the required size of mutants to upset an equilibrium: it shows that an increase in $\hat{\nu}$ from $\hat{\nu}_0$ to $\hat{\nu}_1$ shifts $\tilde{P}_0$ curve downward, while raising $MC(W_t, \hat{\nu})$ without affecting a_t^*. The figure also shows that, when their size exceeds ϵ_1, for either values of $\hat{\nu}$ (i.e., whether $\hat{\nu}_0$ or $\hat{\nu}_1$), a new investment is unprofitable. Moreover, we can verify that $\hat{\nu}_1$ becomes optimal, so that the mutants with this strategy can survive and thrive. In other words, the mutants can effectively upset the equilibrium with smaller population by playing the employment strategy $\hat{\nu}_1$, than by playing $\hat{\nu}_0$.

In contrast, Figure 4-(b) shows that, if $\hat{\nu}$ is below the threshold value $\underline{\nu}$, then the equilibrium output stays higher than P_t so that mutants with no-investment strategy cannot survive. Intuitively, the significant reduction of the current output on the part of mutants by hiring less workers will push up output price. The benefit of this price rise goes to other firms as well. However, they do not bear cost of output reduction as mutants do. As a result, the mutants cannot survive.

Increasing $\hat{\nu}$ beyond $\bar{\nu}_t$ is unprofitable because the increased consumption resulted from the increased employment would spill over to other firms. For the same reason, an increase in $\hat{\zeta}$ will be unprofitable: the increased demand generated by increased investment demand will spill over to the other firms in the form of higher output price.

2. Main Results

To describe the above intuition formally, take a KTE with $\bar{x}_t$ as given. For a given post-entry price P in $\mathcal{E}(\epsilon, (\hat{\nu}, \hat{\zeta}))$, let $E^d(p, \epsilon)$ denote the value (in money term) of aggregate excess demand:

$$E^d(P, \epsilon, \hat{\nu}, \hat{\zeta}) = c_y\big(P_{t-1}\bar{H} + W_t\tilde{N}_t + r_tP_{t-1}K_{t-1}\big) + c_wP_{t-1}K_{t-1} - P(\tilde{Q}_t - \tilde{Z}_t) \tag{35}$$

$$= Y_t - (S_t - S_{t-1}) + c_yW_t\big(\tilde{N}_t(\epsilon, \hat{\nu}) - N_t\big) - P\big\{\tilde{Q}_t(\epsilon, \hat{\nu}) - \tilde{Z}_t(\epsilon, \hat{\zeta})\big\}. \tag{36}$$

The post-entry market clearing price, denoted by $\tilde{P}$, is determined so that $E^d(\tilde{P}, \epsilon, \hat{\nu}, \hat{\zeta}) = 0$. Hence, we have

$$\tilde{P}(\epsilon, \hat{\nu}, \hat{\zeta}) = \frac{c_y(P_{t-1}\bar{H} + W_t\tilde{N}_t(\epsilon, \hat{\nu}) + r_tP_{t-1}K_{t-1}) + c_wP_{t-1}K_{t-1}}{\tilde{Q}_t(\epsilon, \hat{\nu}) - \tilde{Z}_t(\epsilon, \hat{\zeta})} \tag{37}$$

$$= \frac{C_N - \epsilon(\bar{\nu} - \hat{\nu})c_yW_t}{(1-\epsilon)C_R + \epsilon\big\{\hat{\nu}^{1-\alpha} - \max(\hat{\zeta}, 0)\big\}}. \tag{38}$$

Noting by remark 2 that the marginal cost is below the average direct cost if labor-capital ratio ν is less than the cost minimizing level ν_{ct}, we express the above intuition now formally:

Lemma 9. Let $\eta_t > 1$ and ν_0 be defined by $\eta_t\nu_0^{1-\alpha} = 1 - \delta$. Let $\bar{\epsilon}$ be the value of invasion barrier at $\bar{\nu} = \nu_{ct}$. Assume that $\nu_0 < \bar{\nu} \leq \nu_{ct}$. Then, the most destructive strategy of mutants is $(\bar{\nu}, 0)$. For all $\bar{\nu}$, the value of invasion barrier $\hat{\epsilon}$ is increasing in $\bar{H}$. Then, $\hat{\epsilon}$ is increasing in $\bar{\nu}$ if $\eta_t > 1/(1 - \bar{\epsilon})$.

Proof. See Appendix I. □

Lemma 9 says that the amount of salary payments to overhead employees work as a margin of safety that ensures the profitability of investment.

The following result is the counterpart of lemma 9. If $\bar{\nu}_t$ is greater than the cost minimizing employment ratio, $\nu_{c,t}$, the best-responding mutants should choose $\nu_{c,t}$ instead of mimicking the incumbents' ratio $\bar{\nu}_t$ because the incumbents employment ratio $\bar{\nu}_t$ is less attractive than $\nu_{c,t}$ for their purpose of reducing the output price.

Lemma 10. Assume that $\eta_t > 1$ and $\nu_{ct} \leq \bar{\nu}$. Then, the most destructive strategy of mutants is $(\nu_{ct}, 0)$. For all $\bar{\nu}$ and $\bar{\nu}$, the value of invasion barrier $\hat{\epsilon}$ is increasing both in $\bar{H}$ and $\bar{\nu}$.

Proof. See Appendix J □

To obtain more concrete results, suppose that the economy has been in a steady-state equilibrium up until period $t-1$. This implies that the assumption of the previous lemma holds since in any KTE with unemployment, the labor-capital ratio is less than the cost-minimizing ratio.

Proposition 4. Assume that $\bar{H}\{c_w/(1-c_y)\}^{1/(1-\alpha)} < \delta\{(1-\delta)(1-c_y)/c_w-1\}$ and that an economy has been in a steady-state equilibrium up until period $t-1$. Further assume that in -eriod t, an economy is in Keynesian temporary equilibrium with unemployment, i.e., $\bar{\nu}_t \leq \nu_{ss}$. Then the invasion barrier $\hat{\epsilon}$ increases with $\bar{\nu}$ as well as with $\bar{H}$.

Proof. See Appendix Appendix K □

For the case of a steady state equilibrium, the result is summarized by Figure 5. If a post-entry aggregate investment is larger than $\tilde{Z}^*$, the post-entry output price does not fall below the minimum average direct cost, i.e., $\tilde{P}_0 > a_t^*$, thus making zero-investment strategy worse off than incumbents' strategy $\zeta_{ss} = \delta$. This will diminish the population of the mutants. Therefore, for successful invasion, their population must be at least as large as $\hat{\epsilon}$.

The next corollary derives the exact expression of $\hat{\epsilon}$ for a steady state equilibrium. An increase in $\bar{H}$, c_y, or c_w will stabilize an economy in a steady-state equilibrium

Corollary 11. A steady-state equilibrium is robust with degree of $\hat{\epsilon}$ against boundedly rational mutant where $\hat{\epsilon} = c'(c' + \delta)^{1/(1-\alpha)}\bar{H}/\delta$. In particular, if $\bar{H} = 0$, the steady-state equilibrium is unstable, i.e., $\hat{\epsilon} = 0$.

Proof. See the proof of proposition 4. □

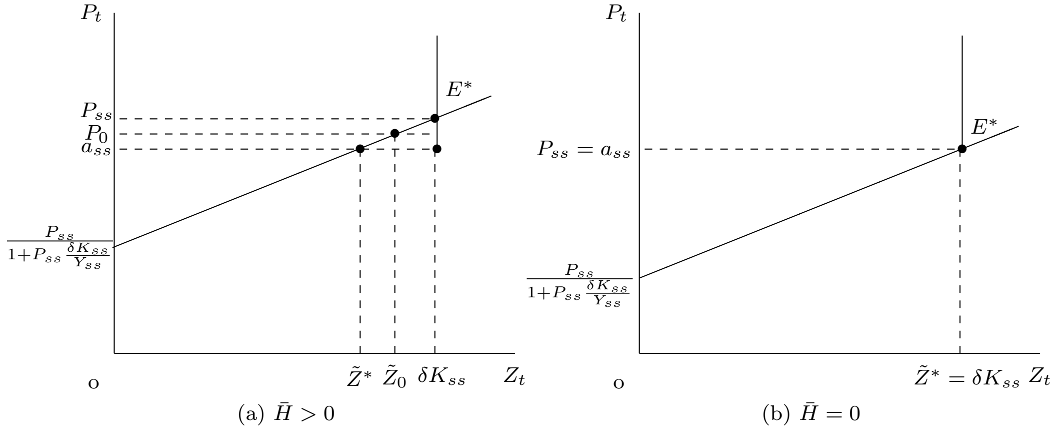

Figure 5: Stability of Steady State Equilibrium

6 Implications of the results

We have demonstrated that a KTE is robust up to an invasion barrier, $\bar{\epsilon}$, against boudedly rational mutants. This means that the equilibrium is robust against *any mutant strategy* as far as the mutant population is smaller than $\bar{\epsilon}$. Conversely, we have found that, if zero-investment mutants enter unemployment Keynesian temporary equilibrium with population size greater than the invasion barrier, the output price plunges, thus reducing the demand price of capital low enough to trigger debt-deflation.

By the way, Fisher argued that only sizable over-indebtedness can trigger debt deflation: "[w]hen over-indebtedness stands alone, that is, does not lead to a fall of prices, [...], the resulting 'cycle' will be far milder and far more regular. [...] But if the over-indebtedness is not sufficiently great to make liquidation thus defeat it self, the situation is different and simpler. It is then more analogous to stable equilibrium; the more the boat rocks the more it will tend to right itself" (Fisher (1933), p. 344). One natural interpretation of zero-investment mutants is those who are forced to stop investment to liquidate the over-indebtedness. With this interpretation, our stability results stated above seem consistent with this Fisher's insight.

7 Conclusion

We have presented a model with demand-constrained firms which undertake investment only when a certain level of profits is expected. Then, we have proved the existence of a unique steady-state equilibrium. Defining Keynesian temporary equilibrium and we have shown that there exist multiple equilibria near the steady-state if compensation to overhead employees is positive. Our boundedly rational firms take the current output price in excess of its average direct cost as a summary of future profits. Uncertainty and unreliable views about future make investment activity as flimsy decision quickly changeable making. In the spirit of evolutionary game theory, we have modeled the changing views about future as mutants and explored the robustness of a temporary Keynesian equilibrium. We have found that a demand deficient equilibrium exists near the steady-state equilibrium if the compensation to overhead employees is positive. Our main result is that the mutant strategy of zero-investment, which may be interpreted as liquidation of over-indebtedness, capsizes an economy with the smallest population of mutants. It then reduces the level of *output price*, hence expected quasi-rents, low enough to trigger debt-deflation process. We have proved that the amount of compensation to overhead employees is related with the robustness of the economy; the larger the amount of payment to overhead employees, the more robust a Keynesian temporary equilibrium becomes. An increase in propensities to consume also enhances the stability but reduces the level of steady-state output.

Regarding the extension of this research, several directions seem interesting. We can incorporate expectation formation about demands and endogenize money supply to build a dynamic model that can explore the issue of dynamic stability and business cycles[34]. Endogenizing safety margins and financial structure of the bank and firms seems also interesting avenues.

[34] See Takahashi and Okada (2013) for this direction.

Appendix A Employment Decision

Figure 1 describes the labor demand decision of a firm under two types of demand constraint denoted by $\bar{q}_t$: a binding constraint, e.g., $\bar{q}_t^b$, and an unbinding constraint, e.g., $\bar{q}_t^u$[35]. In either case, recognizing the difficulty of selling more than $\bar{q}_t$ without reducing the price, our firm perceives a kinked demand curve which consists of two different parts: a horizontal part D_1 and a downward-sloping part D_2[36]. As the figure suggests, a small change in real wage rate may not affect employment decision if a firm is demand-constrained.

Here, we will distinguish price set by individual firm, p_t, and market price, P_t. Now, let us denote the short-run production function by $q_t = F(n_t) \equiv k_{t-1}^{\alpha} n_t^{1-\alpha}$. If the demand constraint is not binding, that is, if

$$P_t \leq \frac{W_t}{F'(\bar{n})},$$

then the optimal amount of employment n^* is n^u. Suppose the opposite is the case:

$$P_t > \frac{W_t}{F'(\bar{n})}.$$

Let ξ denote the right-hand side demand elasticity at $q = \bar{q}_t$. We simply assume here that demand is inelastic at $(P_t, \bar{q}_t)$, thus $\xi \leq 1$[37]. Then, the conditions for the profit maximization are satisfied at $(p, q) = (P_t, \bar{q}_t)$:

$$P_t(1 - \frac{1}{\xi}) \leq \frac{W_t}{F'(\bar{n})}, \tag{39a}$$

$$P_t > \frac{W_t}{F'(\bar{n})}. \tag{39b}$$

Equations (39a) and (39b) say that, when q is at $\bar{q}_t$, the marginal cost is greater than the marginal revenue but smaller than the output price. Thus,

[35]The marginal cost is obtained from $W/F'(n)$ where $F(n)$ is the short-run production function.

[36]This shape of perceived demand curve is consistent with Chick's observation: "the demand curve will become downward-sloping at some point determined by the market demand curve and by single firm's share in the market (Chick (1991), p. 88).

[37]Even if $\eta > 1$, the above conditions can still hold unless the marginal cost, $W_t/F'(n)$, is substantially small relative to the output price.

$n_t^* = \bar{n}$, which implies that the firm plans to produce a quantity equal to the expected demand. This leads to equation (40). Define $\bar{n}_t$ as the amount of employment required to produce $\bar{q}_t$, i.e., $\bar{q}_t = F(\bar{n}_t)$. Further, let n_t^u and q_t^u denote the amount of employment and its corresponding output, respectively, under unbinding demand constraint: hence, $P_t F'(n_t^u) = W_t$ and $F(n_t^u) = q_t^u$. If the demand is binding, the level of employment is dictated by $\bar{q}_t$. Consequently, for given $(P_t, W_t, k_{t-1}, \bar{q}_t)$, the firm will choose an amount of employment, n_t, to maximize the current quasi-rent:

$$\pi_t^q = P_t \min(F(n_t), \bar{q}_t) - W_t n_t. \tag{40}$$

The solution is given by $n_t^* = \min\left(n_t^u, \bar{n}_t\right)$, where $n_t^u = \left((1-\alpha)P_t/W_t\right)^{1/\alpha} k_{t-1}$, and $\bar{n}_t = \left(\bar{q}_t/k_{t-1}^{\alpha}\right)^{1/(1-\alpha)}$ Accordingly, the corresponding short-run output is given by $q_t^* = \min(q_t^u, \bar{q}_t)$, where $q_t^u = \left((1-\alpha)P_t/W_t\right)^{(1-\alpha)/\alpha} k_{t-1}$.

Appendix B Proof of Lemma 1

Proof. From the definition of $\hat{\rho}$, the equivalence between (L1) and (L2) is obvious. Multiplying the both sides of (25) by $\hat{\rho} + \delta$, the second condition is equivalent to

$$\left[P_t \left(\frac{(1-\alpha)U_t}{\alpha W_t}\right)^{1-\alpha} - \frac{(1-\alpha)U_t}{\alpha}\right] = P_{t-1}(\hat{\rho} + \delta) > P_{t-1}(r_t + \delta) = U_t, \tag{41}$$

which is the condition (L3). The inequality of (41) can be expressed as

$$P_t \left(\frac{(1-\alpha)U_t}{\alpha W_t}\right)^{1-\alpha} - \frac{U_t}{\alpha} > 0,$$

whose left hand side is further rewritten as

$$\left(\frac{(1-\alpha)U_t}{\alpha W_t}\right)^{1-\alpha} \left[P_t - \left(\frac{U_t}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\right] = \left(\frac{(1-\alpha)U_t}{\alpha W_t}\right)^{1-\alpha} (P_t - a_t^*),$$

which implies the equivalence between (L2) and (L4). $\square$

Appendix C Proof of Proposition 1

Proof. In this subsection, except for emphasis, we omit the subscript ss that is associated with steady-state. Let $c' = c_w/(1-c_y)$ By (30a) and (30d),

$$\frac{Y}{P} = c'K. \tag{42}$$

By (42) and (30e),

$$Q = (c' + \delta)\, K, \tag{43}$$

hence, $\eta = K/Q = 1/(c' + \delta)$. Substituting (30a) into (43) yields

$$K = (c' + \delta)^{-\frac{1}{1-\alpha}} \bar{N}, \tag{44}$$

hence, $\nu = \bar{N}/K = (c' + \delta)^{1/(1-\alpha)}$. Plugging (44) into (43), we have

$$Q = (c' + \delta)^{-\frac{\alpha}{1-\alpha}} \bar{N}. \tag{45}$$

Substituting (44) into (42) yields

$$\frac{Y}{P} = C = c'(c' + \delta)^{-\frac{1}{1-\alpha}} \bar{N}. \tag{46}$$

We see that K, Q, $\frac{Y}{P}$, and C are uniquely determined and independent of $\bar{H}$. Using (30b) and (44), we see that

$$\frac{W}{P} = \frac{(1-\alpha)(r+\delta)}{\alpha}(c' + \delta)^{-\frac{1}{1-\alpha}}. \tag{47}$$

By the way, substiting (46) and (44) into (30c), we get

$$c'(c' + \delta)^{-\frac{1}{1-\alpha}} = \bar{H} + \frac{W}{P} + r(c' + \delta)^{-\frac{1}{1-\alpha}}, \tag{48}$$

which reduces to

$$(c' + \delta)\{1 - \bar{H}(c' + \delta)^{\frac{\alpha}{1-\alpha}}\} = \frac{r+\delta}{\alpha}. \tag{49}$$

Substituting (47) into (48) and arranging terms, we get

$$\frac{r}{\alpha} + \frac{(1-\alpha)\delta}{\alpha} = c' - (c' + \delta)^{\frac{1}{1-\alpha}} \bar{H}, \tag{50}$$

which implies that r is decreasing in $\bar{H}$. Obtain $\bar{H}' = (c' + \delta - \delta/\alpha)(c' + \delta)^{1/(1-\alpha)}$ as the value of $\bar{H}$ that gives $r = 0$ in (50). If $\bar{H}' > 0$, by choosing a $\bar{H} \in [0, \bar{H}']$, r is uniquely detemined. Since $\bar{M}$ is exogenously given, (30f) gives Y, implying P is uniquely determined. This in turn gives W. □

Appendix D Proof of Lemma 5

Proof. In this subsection, we continue to omit the subscript, ss, that is associated with steady-state. We must compute the average direct cost. By (22), the direct cost is expressed as

$$Wn + Uk = \frac{Wn}{1-\alpha}.$$

The production function reduces to

$$q = k^{\alpha} n^{1-\alpha} = \left(\frac{k}{n}\right)^{\alpha} n = \left(\frac{\alpha W}{(1-\alpha)U}\right)^{\alpha} n.$$

Therefore, we obtain

$$\begin{aligned} a^* \equiv \frac{Wn + Uk}{q} &= \frac{Wn}{\left(\frac{\alpha W}{(1-\alpha)U}\right)^{\alpha} (1-\alpha)n} \\ &= \left(\frac{U}{\alpha}\right)^{\alpha} \left(\frac{W}{1-\alpha}\right)^{1-\alpha}. \end{aligned} \tag{51}$$

By (8) and $K_t = K_{t-1}$, in the steady-state, $Z = \delta K$. This implies that, by (16) and $L_t = L_{t-1}$, $\Pi = 0$. Therefore, by (15) $PQ = P(C + Z) = PN_f \bar{H} + WN_f + UK = P\bar{N}\bar{H} + a^* Q$, which leads to

$$P\{1 - \bar{H}(c' + \delta)^{\alpha/(1-\alpha)}\} = a^* \tag{52}$$

from (45). □

Appendix E Proof of Proposition 2

Proof. It follows from (49) that

$$c' + \delta = \bar{H}(c' + \delta)^{\frac{1}{1-\alpha}} + \frac{r + \delta}{\alpha}. \tag{53}$$

Solving (50) for r, we get

$$r = \alpha\{c' - \frac{1-\alpha}{\alpha}\delta - \bar{H}(c' + \delta)^{\frac{1}{1-\alpha}}\}. \tag{54}$$

Diffrentiating (54) with respect to c' and using equation (54), we see

$$
\begin{aligned}
\frac{\partial r}{\partial c'} &= \alpha - \frac{\alpha}{1-\alpha} \frac{\bar{H}(c'+\delta)^{\frac{1}{1-\alpha}}}{c'+\delta} \\
&= \alpha + \frac{r - \alpha c' + (1-\alpha)\delta}{(1-\alpha)(c'+\delta)} \\
&= \alpha + \frac{r + \delta - \alpha(c'+\delta)}{(1-\alpha)(c'+\delta)} \\
&= \alpha - \frac{\alpha}{1-\alpha} + \frac{r+\delta}{(1-\alpha)(c'+\delta)} \\
&= \frac{1}{1-\alpha}\left(-\alpha^2 + \frac{r+\delta}{c'+\delta}\right) > 0 \qquad (55)
\end{aligned}
$$

where the last inequality follows from (53), i.e., $(r+\delta)/(c'+\delta) > \alpha$. The inequality (55) implies $\partial Y/\partial c' > 0$ by (30f). This in turn leads to $\partial P/\partial c'$ since real income decreases with c'. Using (53) equation (47) can be rewritten as

$$
\frac{W}{P} = (1-\alpha)\frac{r+\delta}{\alpha}\left(\bar{H} + \frac{r+\delta}{\alpha}\right)^{-\frac{1}{1-\alpha}}. \qquad (56)
$$

Differentiating (47) with respect to c' yields

$$
\begin{aligned}
\frac{\partial(W/P)}{\partial c'} &= \frac{1-\alpha}{\alpha}\left\{\left(\bar{H} + \frac{r+\delta}{\alpha}\right)^{-\frac{1}{1-\alpha}} - \frac{1}{1-\alpha}(r+\delta)\left(\bar{H} + \frac{r+\delta}{\alpha}\right)^{-\frac{1}{1-\alpha}-1}\right\} \\
&= \frac{1-\alpha}{\alpha}\left(\bar{H} + \frac{r+\delta}{\alpha}\right)^{-\frac{1}{1-\alpha}-1}\left[\bar{H} + \frac{r+\delta}{\alpha} - \frac{r+\delta}{1-\alpha}\right] \\
&= \frac{1-\alpha}{\alpha}\left(\bar{H} + \frac{r+\delta}{\alpha}\right)^{-\frac{1}{1-\alpha}-1}\left[\bar{H} + (r+\delta)\frac{1-2\alpha}{\alpha(1-\alpha)}\right],
\end{aligned}
$$

which is positive if $\alpha < 1/2$. Thus, W_{ss} rises faster than P_{ss} does. By (53), a rise in $\bar{H}$ lowers r, thus reducing Y by (30f). Since Y/P remains the same, P_{ss} must decline. Since W/P declines, W_{ss} drops even more than P_{ss} does. □

Appendix F Proof of Lemma 6

Proof. Dividing the numerator and denominator of (13) by K_{t-1}, we have

$$
P(\nu_t) = \frac{c_y W_{ss}\nu_t + E_t}{\nu_t^{1-\alpha} - \max\{\eta_{ss}\nu_t^{1-\alpha} - (1-\delta), 0\}}, \qquad (57)
$$

where $E_t = P_{t-1}\{c_y(\bar{H} + r_t K_{t-1}) + c_w K_{t-1}\}$

$$\begin{aligned}
P'(\nu) &= \frac{c_y W C_R}{C_R^2} - \frac{C_N\{(1-\alpha)(1-\eta)\nu^{-\alpha}\}}{C_R^2} \\
&= \frac{1}{C_R}\{c_y W - p(1-\alpha)(1-\eta)\nu^{-\alpha}\}
\end{aligned}$$

$$\begin{aligned}
MC' &= \frac{W\alpha\nu^{\alpha-1}}{1-\alpha} \\
&= MC\frac{\alpha}{\nu}.
\end{aligned}$$

$$\begin{aligned}
P'(\nu) - MC' &= \frac{1}{C_R\nu}\{c_y W\nu - P(1-\alpha)(1-\eta)\nu^{1-\alpha} - MC\alpha C_R\} \\
&= \frac{1}{C_R\nu}[c_y W\nu - P(1-\alpha)(1-\eta)\nu^{1-\alpha} - P\alpha\{(1-\eta)\nu^{1-\alpha} + (1-\delta)\}] \\
&= \frac{1}{C_R\nu}[c_y W\nu + P(1-\alpha)(\eta-1)\nu^{1-\alpha} - \alpha C_N] \\
&= \frac{1}{C_R\nu}[c_y W\nu + P(1-\alpha)(\eta-1)\nu^{1-\alpha} - \alpha(c_y W\nu + E/K)] \\
&= \frac{1}{C_R\nu}[(1-\alpha)c_y W\nu - \alpha E/K + P(1-\alpha)(\eta-1)\nu^{1-\alpha}] \qquad (58)
\end{aligned}$$

By the way,

$$\begin{aligned}
(1-\alpha)c_y W\nu - \alpha E/K &= (1-\alpha)(c_y W\nu + E/K) - (1-\alpha)E/K - \alpha E/K \\
&= (1-\alpha)C_N - E/K \qquad (59)
\end{aligned}$$

$$\begin{aligned}
P(1-\alpha)(\eta-1)\nu^{1-\alpha} &= -(1-\alpha)p\{(1-\eta)\nu^{1-\alpha} + (1-\delta)\} + (1-\alpha)(1-\delta)p \\
&= -(1-\alpha)C_N + P(1-\alpha)(1-\delta) \qquad (60)
\end{aligned}$$

Substituting (59) and (60) into (58) yields

$$P'(\nu) - MC'(\nu) = \frac{1}{PC_R\nu}\left((1-\alpha)(1-\delta) - \frac{E}{PK}\right). \qquad (61)$$

Since the economy has stayed in a SSE until period $t-1$, we see

$$\begin{aligned}
\frac{E_t}{P_{t-1}K_{t-1}} &= \frac{E_{ss}}{P_{ss}K_{ss}} \\
&= \frac{Y_{ss}}{P_{ss}K_{ss}} - c_y\frac{W_{ss}}{P_{ss}}\frac{N_f}{K_{ss}}, \qquad (62)
\end{aligned}$$

which is, by (46) and (47), reduced to

$$\begin{aligned}\frac{E_t}{P_{t-1}K_{t-1}} &= \frac{N_f}{K}(c'+\delta)^{-\frac{1}{1-\alpha}}\left(c' - c_y\frac{(1-\alpha)(r+\delta)}{\alpha}\right)\\ &= c' - c_y\frac{(1-\alpha)(r+\delta)}{\alpha},\end{aligned}$$

where the last equality follows from (44). By (46) and (47),

$$\begin{aligned}\frac{E}{PK} &= \frac{C}{K} - c_y\frac{W}{P}\frac{N}{K}\\ &= c' - c_y\frac{(1-\alpha)(r+\delta)}{\alpha}. \qquad (63)\end{aligned}$$

Let $\delta' = \delta + c_y/(1-c_y)\bar{H}(c'+\delta)^{1/(1-\alpha)}$. Using (53), we can express (63) as

$$\begin{aligned}\frac{E}{PK} &= c' - c_y(1-\alpha)\{c'+\delta-\bar{H}(c'+\delta)^{\frac{1}{1-\alpha}}\}\\ &= c' - c_y(1-\alpha)c' + \delta + c_y(1-\alpha)\bar{H}(c'+\delta)^{\frac{1}{1-\alpha}}\\ &= c' - (1-\alpha)(c'+\delta) + (1-\alpha)\{(1-c_y)(c'+\delta) + c_y\bar{H}(c'+\delta)^{\frac{1}{1-\alpha}}\}\\ &= c' - (1-\alpha)(c'+\delta) + (1-\alpha)(1-c_y)(c'+\delta')\end{aligned}$$

Therefore,

$$\begin{aligned}(1-\alpha)(1-\delta) - \frac{E}{PK} &= (1-\alpha)(1-\delta) - \{\alpha c' - (1-\alpha)\delta\} - (1-\alpha)(1-c_y)(c'+\delta')\\ &= 1-\alpha-\alpha c' - (1-\alpha)(1-c_y)(c'+\delta')\\ &= (1-\alpha)(1-c_y)\Big(\frac{1-\alpha-\alpha c'}{(1-\alpha)(1-c_y)} - c' - \delta'\Big).\end{aligned}$$

Hence, $P'(\bar{\nu}) > MC'(\bar{\nu})$ if and only if

$$\delta' < \frac{1-\alpha-\alpha c' - c'(1-\alpha)(1-c_y)}{(1-\alpha)(1-c_y)}$$

Substituting $\bar{H}(c' + \delta)^{\frac{1}{1-\alpha}} = c' + \delta - \frac{r+\delta}{\alpha}$ into the above expression, the inequality is reduced to

$$\begin{aligned} c_y\Big\{(c'+\delta) - \frac{r+\delta}{\alpha}\Big\} < 1 - \delta - \frac{c'}{1-\alpha} + c_y(c'+\delta) \\ 1 + c_y\frac{r+\delta}{\alpha} > \delta + \frac{c'}{1-\alpha} \\ 1 + c_y\frac{r+\delta}{\alpha} + \frac{\alpha}{1-\alpha}\delta > \frac{c'+\delta}{1-\alpha} \\ (1 - \alpha + \frac{1-\alpha}{\alpha} r c_y) + (\frac{1-\alpha}{\alpha} c_y + \alpha)\delta > c' + \delta. \end{aligned}$$

Therefore, a sufficient condition for the right hand side of (61) to be positive is that

$$\frac{1-\alpha}{\alpha} c_y + \alpha > 1$$

and

$$1 - \alpha > c'.$$

The former inquality holds if $c_y > \alpha$ since the weighted sum of c_y and α, i.e., $(1-\alpha)c_y + \alpha\alpha > \alpha$, is always greater than α. □

Appendix G Proof of Lemma 7

Proof. In what follows, ν is used to represent $\bar{\nu}$. First, it is verifed that, for ν less than ν_δ, the output price decreases as ν increases if the price is greater than the marginal cost. For $\nu < \nu_\delta$, $\zeta = 0$, hence, by (57), $P(\nu) = (c_y W_{ss}\nu + E_{ss})/\nu^{1-\alpha}$ where E_{ss} denote the nominal consumption out of non-wage income per unit of capital goods in the steady state, i.e., $E_{ss} = P_{ss}\{c_y(\bar{H} + r_{ss}K_{ss}) + c_w K_{ss}\} = P_{ss}C_{ss}/K_{ss} - c_y W_{ss}\nu_{ss}$.

$$\begin{aligned} \frac{dP}{d\nu} &= \frac{c_y W_{ss}\nu^{1-\alpha} - (c_y W_{ss}\nu + E_{ss})(1-\alpha)\nu^{-\alpha}}{\nu^{2(1-\alpha)}} \\ &= \frac{W_{ss}}{\nu^{1-\alpha}}\{c_y - P/MC(W_{ss}, \nu)\}, \end{aligned}$$

which is negative if $P > MC(W_{ss}, \nu)$.

Next, let's obtain the level of $\bar{H}$ that equates $P(\nu_\delta)$ to a^*. Since E_{ss} includes the compensation to overhead employees $P(\nu_t)$ is increasing in $\bar{H}$

for all ν_t. Notice that $\nu_\delta^{1-\alpha} = (1-\delta)/\eta_{ss}$. Moreover, $\eta_{ss} = \{\alpha W_{ss}/((1-\alpha)U_{ss})\}^{1-\alpha} = (\alpha/U_{ss})(1/A)(U_{ss}/\alpha)^\alpha(W_{ss}/(1-\alpha))^{1-\alpha} = \alpha a^*_{ss}/U_{ss}$. Further, we see that

$$\begin{aligned} c_y W_{ss}\nu_\delta + E_{ss} &= c_y W_{ss}\nu_{ss} + E_{ss} - c_y W_{ss}(\nu_{ss} - \nu_\delta) \\ &= P_{ss}c' - c_y W_{ss}\{(c'+\delta)^{1/(1-\alpha)} - \nu_\delta\} \end{aligned}$$

Therefore, by (31) and (56), the condition $P(\nu_\delta) = a^*$ can be expressed as follows:

$$P(\nu_\delta) = \frac{c_y W_{ss}\nu_\delta + E_{ss}}{\nu_\delta^{1-\alpha}} = a^*_{ss} \tag{64}$$

$$\frac{P_{ss}[c' - c_y\frac{1-\alpha}{\alpha}(r_{ss}+\delta)\{1-(1-\delta)^{1/(1-\alpha)}\}]}{(1-\delta)(c'+\delta)} = \frac{\eta_{ss}U_{ss}}{\alpha} \tag{65}$$

$$\frac{1}{1-\delta}[c' - c_y\frac{1-\alpha}{\alpha}(r_{ss}+\delta))\{1-(1-\delta)^{1/(1-\alpha)}\}] = \frac{r_{ss}+\delta}{\alpha}. \tag{66}$$

Using (49), we get

$$\{c'+\delta-\bar{H}_\delta(c'+\delta)^{1/(1-\alpha)}\}\left(1+\frac{c_y(1-\alpha)\{1-(1-\delta)^{1/(1-\alpha)}\}}{1-\delta}\right) = \frac{c'}{1-\delta}, \tag{67}$$

implying that

$$\bar{H}_\delta = \frac{c'+\delta-c'/\{\Delta(1-\delta)\}}{(c'+\delta)^{1/(1-\alpha)}},$$

where $\Delta = 1+c_y(1-\alpha)\{1-(1-\delta)^{1/(1-\alpha)}\}/(1-\delta)$. For a given ν, the output price is increasing in $\bar{H}$. Hence, $P(\nu) > a^*_{ss}$.

By (13), the following equality holds:

$$F = c_y W_{ss}\nu' + E_{ss} - a^*_{ss}\Big((1-\eta_{ss})\nu'^{1-\alpha} + 1 - \delta\Big) = 0,$$

$$\frac{\partial F}{\partial \nu'} = c_y W_{ss} - a_{ss}(1-\eta_{ss})(1-\alpha)\nu'^{1-\alpha} > 0.$$

The result follows because E_{ss} depends on $\bar{H}$ positively. □

Appendix H Explanation of Remark 5

The total productive capital stocks of mutants and an incumbent in period(t+1) are respectively given by $\epsilon\{\hat{\zeta} + (1-\delta)\}K_{t-1}$, and $(1-\epsilon)\{\bar{\zeta}_t + (1-\delta)\}K_{t-1}$. Choosing $\nu_{c,t}$, the firms expect to determine the amounts of employment to be combined with these capital stocks. Since the aggregate demand is equal to aggregate output, the overall demand in period $t+1$ will be $\nu_{c,t}^{1-\alpha}[\epsilon\{\hat{\zeta} + (1-\delta)\} + (1-\epsilon)\{\bar{\zeta}_t + (1-\delta)\}]K_{t-1}$ By Assumption 1, the aggregate demand is distributed among the firms in proportion to their capital stock in period $t-1$[38] Therefore, the mutants as a whole expect to receive the following demand: $\epsilon\nu_{c,t}^{1-\alpha}[\epsilon\{\hat{\zeta}+(1-\delta)\}+(1-\epsilon)\{\bar{\zeta}_t+(1-\delta)\}]K_{t-1}$, which is larger than their collective output $\epsilon\nu_{c,t}^{1-\alpha}\big(\hat{\zeta} + (1-\delta)\big)$ if $\hat{\zeta} < \bar{\zeta}_t$.

Appendix I Proof of Lemma 9

Proof. Assume that $\bar{\nu}_t \leq \nu_{c,t}$.

Step 1

We first claim that the strategy pair of mutants that can upset a Keynesian temporary equilibrium most effectively, i.e., with its smallest population size $\hat{\epsilon}$, is $(\hat{\nu}, \hat{\zeta}) = (\bar{\nu}_t, 0)$. We verify this claim for two cases depending on the value of $\hat{\nu}$: (i) $\hat{\nu} > \bar{\nu}_t$ and (ii) $\hat{\nu} \leq \bar{\nu}_t$.

<u>*Case (i)*</u> $\hat{\nu} > \bar{\nu}_t$

This employment strategy cannot be optimal. By Remark 3 the mutants are producing output beyond their demand constraint, i.e., $\hat{\nu}^{1-\alpha} > \tilde{\lambda}$, so that, regardless of the value of $\hat{\zeta}$, they can cut wage bill without reducing revenue.

<u>*Case (ii)*</u> $\hat{\nu} \leq \bar{\nu}_t$

We want to show that if $\hat{\nu} < \bar{\nu}_t$, then the investment ratio of a successful mutant must be $\hat{\zeta} = 0$. To show this formally, assume, on the contrary, that

[38] To estimate period $t+1$ demand, we use period $t-1$ capital in stead of period t capital. This assumption attempts to capture some inertia involved in individual demand, for example, long-term customer relationships. One may think that there is some time inconsistency: individual demand in both periods t and $t+1$ depend on k_{t-1}. We could have alternatively assumed that the aggregate demand in period t is distributed among individual firms based on k_{t-2}. Since in period $t-1$ all the firms behaved identically, hence $\zeta_{t-1}(\theta) = \zeta_t\ \forall\theta$ in particular. Therefore, $k_{t-2}(\theta)/K_{t-2} = k_{t-1}(\theta)/K_{t-1}$. Hence the alternative assumption and the current one are equivalent with each other, hence the current assumption is innocuous.

a successful mutant, best responding to $\mathcal{E}(\epsilon, (\hat{\nu}, \hat{\zeta}))$, has a strategy pair $(\hat{\nu}, \hat{\zeta})$ such that $\hat{\nu} < \bar{\nu}_t$ as well as $\hat{\zeta} > 0$. For a positive $\hat{\zeta}$ to be a best response, the output price, $\tilde{p}$, must bring the value of quasi-rents large enough to ensure

$$P^K(\tilde{P}, W_t, U_t, r_t) = \sum_{s=1}^{\infty} \frac{(1-\delta)^{s-1}}{(1+r_t)^s} \left[\tilde{P}\Big(\frac{(1-\alpha)U_t}{\alpha W_t}\Big)^{1-\alpha} - \Big(\frac{1-\alpha}{\alpha} U_t\Big) \right] \geq P_{t-1}.$$

By Corollary 2, this inequality implies that the price must exceed the minimum average cost, a_t^*, that coincides with the marginal cost at $\nu = \nu_{c,t}$, i.e., $\tilde{P} > MC(W_t, \nu_{c,t}) = W_t \nu_{c,t}^{\alpha}/(1-\alpha)$. Since $\hat{\nu} < \bar{\nu}_t < \nu_{c,t}$ and $MC(W_t, \nu)$ is increasing in ν, we see $\tilde{p} > MC(W_t, \hat{\nu}) = W_t \hat{\nu}^{\alpha}/(1-\alpha)$. In addition, by remark 4, the inequality $\hat{\nu} < \bar{\nu}_t$ implies that the demand constraint is not binding, i.e., $\hat{\nu}^{1-\alpha} < \tilde{\lambda}$. Therefore, remark 1 implies that the mutants have an incentive to increase employment from $\hat{\nu} k_{t-1}$, contradicting the assumption that $(\hat{\nu}, \hat{\zeta})$ best responds to $\mathcal{E}(\epsilon, (\hat{\nu}, \hat{\zeta}))$. Hence, it must be the case that $P^K(\tilde{p}, W_t, U_t, r_t) < P_{t-1}$, thereby $\hat{\zeta} = 0$. Thus, given $\hat{\nu} < \bar{\nu}_t$, we can narrow down the range of best responding mutants to those with $\hat{\zeta} = 0$.

Suppose the strategy pair of mutants is $(\hat{\nu}, 0)$. Setting $\hat{\zeta} = 0$ in (38), the output price after the entry of zero-investment mutants, which is denoted by $\tilde{P}_0(\epsilon, \hat{\nu})$, is expressed as:

$$\tilde{P}_0(\epsilon, \hat{\nu}) = \frac{C_N - \epsilon(\bar{\nu} - \hat{\nu}) c_y W_t}{(1-\epsilon)C_R + \epsilon \hat{\nu}^{1-\alpha}}. \tag{68}$$

Differentiating (68) with respect to ϵ yields

$$\begin{aligned}
\frac{\partial \tilde{P}_0(\epsilon, \hat{\nu})}{\partial \epsilon} &= \frac{-(\bar{\nu} - \hat{\nu}) c_y W_t \{(1-\epsilon)C_R + \epsilon \hat{\nu}^{1-\alpha}\} - \{C_N - \epsilon(\bar{\nu} - \hat{\nu}) c_y W_t)\}(-C_R + \hat{\nu}^{1-\alpha})}{\{(1-\epsilon)C_R + \epsilon \hat{\nu}^{1-\alpha}\}^2} \\
&= \frac{C_R\{-(\bar{\nu} - \hat{\nu}) c_y W_t + C_N\} - \hat{\nu}^{1-\alpha}(\bar{\nu} - \hat{\nu}) c_y W_t \epsilon - \hat{\nu}^{1-\alpha}(C_N - \epsilon(\bar{\nu} - \hat{\nu}) c_y W_t)}{\{(1-\epsilon)C_R + \epsilon \hat{\nu}^{1-\alpha}\}^2} \qquad (69) \\
&= \frac{C_R\{C_N - c_y W_t \bar{\nu} + c_y W_t \hat{\nu}\} - \hat{\nu}^{1-\alpha} C_N}{\{(1-\epsilon)C_R + \epsilon \hat{\nu}^{1-\alpha}\}^2} \\
&= \frac{C_R(E_t/K_{t-1} + c_y W_t \hat{\nu}) - C_N \hat{\nu}^{1-\alpha}}{\Big((1-\epsilon)C_R + \epsilon \hat{\nu}^{1-\alpha}\Big)^2}, \qquad (70)
\end{aligned}$$

where $E_t = P_{t-1}\{c_y(\bar{H} + r_t K_{t-1}) + c_w K_{t-1}\}$.

Let $\underline{\nu}$ be the value of $\hat{\nu}$ that satisfies $\partial \tilde{P}_0(\epsilon, \hat{\nu})/\partial \epsilon = 0$[39]: hence, $\tilde{P}_0(\epsilon, \hat{\nu})\underline{\nu}^{1-\alpha} = c_y W_t \underline{\nu} + E_t/K_{t-1}$. Thus, $\tilde{P}_0(\epsilon, \hat{\nu})$ is increasing in ϵ for $\hat{\nu} \in (0, \underline{\nu})$ and decreasing in ϵ for $\hat{\nu} \in (\underline{\nu}, \bar{\nu}_t)$ We also note that for any ϵ, $\tilde{P}_0(\epsilon, \underline{\nu}) = P_t$. (See, Figure 4)

Using this threshold value $\underline{\nu}$, *Case (ii)* is subdivided into two cases.

Case (ii-1) $\hat{\nu} \in (\underline{\nu}, \bar{\nu}_t]$ and $\hat{\zeta} = 0$

We want to show below that the entry barrier becomes smallest when mutant employment strategy $\hat{\nu} = \bar{\nu}$, namely, any mutant with $\hat{\nu}$ in this interval is not as effective as those with $(\bar{\nu}_t, 0)$ to upset the KTE. In other words, we can lower the threshold value $\hat{\epsilon}$ by increasing $\hat{\nu}$. To get intuition, compare two diffrent values of $\hat{\nu}$ in Figure 4-(a). We show below that, for a given $\hat{\epsilon}$, as far as the post-entry price, $\tilde{P}_0(\epsilon, \hat{\nu})$, exceeds marginal cost $MC(W_t, \hat{\nu})$, the mutants with employment strategy $\hat{\nu}$ can increase profit by increasing employment without violating demand constraint. As a result, if $\hat{\nu}_0$ mutants enter with population ϵ_0, post-entry output price is greater than the marginal cost, so that they cannnot survive. The population size, ϵ, that ensures $\hat{\nu}$-mutant's survival is given by the intersection of $\tilde{P}_0(\epsilon, \hat{\nu})$ and $MC(W_t, \hat{\nu})$. By Corollary 2, $P^K(a_t^*, W_t, U_t, r_t) = P_{t-1}$. Further, since $a_t(\nu)$ achieves its minimum a_t^* at $\nu = \nu_{c,t}$, by remark 2, $a_t^* > MC(W_t, \hat{\nu})$ for $\hat{\nu} < \nu_{c,t}$. If the output price falls below a_t^*, firms would stop new investment even if sufficient demand is expected. When the output price falls further down to $\tilde{P}_0(\epsilon, \hat{\nu}) = MC(W_t, \hat{\nu})$, $\hat{\nu}$ becomes the profit maximizing level of employment ratio. Differentiating (68) with respect to $\hat{\nu}$ yields

$$\begin{aligned}\frac{\partial \tilde{P}_0(\epsilon, \hat{\nu})}{\partial \hat{\nu}} &= \frac{\epsilon c_y W_t\{(1-\epsilon)C_R + \epsilon \hat{\nu}^{1-\alpha}\} - (C_N - \epsilon(\bar{\nu} - \hat{\nu})c_y W_t)\epsilon(1-\alpha)\hat{\nu}^{-\alpha}}{\{(1-\epsilon)C_R + \epsilon \hat{\nu}^{1-\alpha}\}^2} \\ &= \frac{\epsilon c_y W_t - \tilde{P}_0(\epsilon, \hat{\nu})\epsilon(1-\alpha)\hat{\nu}^{-\alpha}}{(1-\epsilon)C_R + \epsilon \hat{\nu}^{1-\alpha}\}} \\ &= \frac{\epsilon(1-\alpha)\hat{\nu}^{-\alpha}\big(c_y MC(W_t, \hat{\nu}) - \tilde{P}_0(\epsilon, \hat{\nu})\big)}{C_R - \epsilon\big(C_R - \hat{\nu}^{1-\alpha}\big)},\end{aligned} \tag{71}$$

[39] We see that the numerator of (70) is strictly decreasing in $\hat{\nu}$. The partial derivative of the numerator of (70) with respect to $\hat{\nu}$ can be expressed as $C_R(c_y W_t - C_N(1-\alpha)\hat{\nu}^{1-\alpha}) = C_R(1-\alpha)\hat{\nu}^{-\alpha}\big(c_y MC(W_t, \hat{\nu}) - C_N/CR\big) = C_R(1-\alpha)\hat{\nu}^{-\alpha}\big(c_y MC(W_t, \hat{\nu}) - P_t\big) < 0$ since $P_t > MC$ in KTE. Moreover, the value of the numerator is $C_R E_t/K_{t-1} > 0$ for $\hat{\nu} = 0$ and by inspecting the numerator of (69) $C_N(C_R - \hat{\nu}^{1-\alpha}) = -C_N \bar{\zeta}_t < 0$ for $\hat{\nu} = \bar{\nu}_t$. Consequently, there is a unique threshold value, $\underline{\nu}$, strictly between 0 and ν_t such that $\partial \tilde{P}_0/\partial \epsilon = 0$.

which is negative as far as $\tilde{P}_0(\epsilon, \hat{\nu}) > MC(W_t, \hat{\nu}) = W_t \hat{\nu}^\alpha/(1-\alpha)$. We know that a group of mutants of population ϵ will upset the KTE if the strategy of mutants, $(\hat{\nu}, 0)$, are best responding to $\mathcal{E}(\epsilon, \hat{\nu}, 0)$. This is true if $\tilde{P}_0(\epsilon, \hat{\nu})$ coincides with a_t^* since, as noted above, for $\hat{\nu} < \nu_{c,t}$, a_t^* is always greater than $MC(W_t, \hat{\nu})$. Define the difference between these prices by $F(\epsilon, \hat{\nu})$. Hence, this critical level of population size, $\hat{\epsilon}$, is given by

$$F(\hat{\epsilon}, \hat{\nu}) = \tilde{P}_0(\hat{\epsilon}, \hat{\nu}) - a_t^* = 0. \tag{72}$$

Since $\tilde{P}_0(\hat{\epsilon}, \hat{\nu}) = a_t^*$ in $\mathcal{E}(\hat{\epsilon}, (\hat{\nu}, 0))$, $\hat{\zeta} = 0$ is a best response. Thus, for given $\hat{\nu}$, $\hat{\epsilon})$ is a candidate of the smallest ϵ. As noted above, for $\tilde{P}_0(\epsilon, \hat{\nu}) > \tilde{P}_n(\hat{\nu})$, we have

$$\frac{d\hat{\epsilon}}{d\hat{\nu}} = -\frac{\partial \tilde{P}_0(\epsilon, \hat{\nu})/\partial \hat{\nu}}{\partial \tilde{P}_0(\epsilon, \hat{\nu})/\partial \epsilon} < 0.$$

Thus, we can make the threshold value of ϵ smaller by increasing $\hat{\nu}$ so far as $\hat{\nu} < \bar{\nu}_t$. Therefore, the smallest population size of successful mutants is achieved when $\hat{\nu} = \bar{\nu}_t$ in the interval $\hat{\nu} \in (\underline{\nu}, \bar{\nu}_t]$. Let $\bar{\epsilon} \equiv \hat{\epsilon}(\bar{\nu}_t)$, the mutants are best responding not only with respect to investment but also to employment since the mutants are selecting the level of output just equal to the demand constraint, $\bar{\nu}^{1-\alpha} = \lambda_t$, although the output price exceeds the marginal cost, i.e., $\tilde{P}_0(\hat{\epsilon}, \bar{\nu}_t) = a_t^* > W_t \bar{\nu}_t^\alpha/(1-\alpha)$.

<u>*Case (ii-2)*</u> $\hat{\nu} < \underline{\nu}$ and $\hat{\zeta} = 0$

By assumption $\hat{\nu} < \underline{\nu} < \bar{\nu}_t$, so that $\tilde{P}_0$ is increasing in ϵ by (70) (Figure 4-(b)). Thus, $\forall \epsilon$,

$$\tilde{P}_0(\epsilon, \hat{\nu}) > P_t \tag{73}$$

By the definition of KTE, we know that

$$P_t > MC(W_t, \bar{\nu}_t) = MC(W_t, \bar{\nu}_t) \tag{74}$$

Since $MC(W_t, \hat{\nu})$ is increasing in $\hat{\nu}$, (73) together with (74) imply that

$$\tilde{P}_0(\epsilon, \hat{\nu}, > P_t > MC(W_t, \bar{\nu}_t) > MC(W_t, \hat{\nu}_t). \tag{75}$$

This says the post-entry output price exceeds post-entry marginal cost. Further, by Remark 4, $\hat{\nu}^{1-\alpha} < \tilde{\lambda}(\epsilon, \hat{\nu})$. This means the demand constraint is not binding.

Similarly, by the property of KTE, we see $P_t > a_t^*$. Thus, by (75), we also have

$$\tilde{P}_0(\epsilon, \hat{\nu}) > a_t^*. \tag{76}$$

Consequently, $\forall\epsilon \in (0,1)$ and $\forall\hat{\nu} < \underline{\nu}$ the mutants are not best responding to post-entry environment $\mathcal{E}(\epsilon, \hat{\nu}, 0)$ in investment as well as employment decisions.

From *Case (ii-1)* and *Case (ii-2)*, we have shown that mutants with a strategy pair, $(\hat{\nu}, \hat{\zeta}) = (\bar{\nu}_t, 0)$, can upset a KSE with the smallest population $\hat{\epsilon}$. In other words, The strategy, $(\bar{\nu}_t, 0)$ best responds to $\mathcal{E}(\hat{\epsilon}, (\bar{\nu}_t, 0))$.

Step 2

Substituting $\hat{\nu} = \bar{\nu}$ and $\hat{\zeta} = 0$ into (38), we have

$$\tilde{P}(\epsilon, \bar{\nu}, 0) = \frac{c_y(P_{t-1}\bar{H}/K_{t-1} + W_t\bar{\nu} + r_tP_{t-1}) + c_wP_{t-1}}{\bar{\nu}^{1-\alpha} - (1-\epsilon)\{\eta_t\bar{\nu}^{1-\alpha} - (1-\delta)\}}. \tag{77}$$

Further, setting $\tilde{P}(\epsilon, \bar{\nu}, 0) = a_t^*$ by (72), invasion barrier $\hat{\epsilon}$ must satisfy that

$$\begin{aligned} c_y(P_{t-1}\bar{H}/K_{t-1} &+ W_t\bar{\nu} + r_tP_{t-1}) + c_wP_{t-1} \\ &= a_t^*\Big(\bar{\nu}^{1-\alpha} - (1-\hat{\epsilon})\{\eta_t\bar{\nu}^{1-\alpha} - (1-\delta)\}\Big) \qquad (78)\\ &= a_t^*\Big(\{1 - (1-\hat{\epsilon}\eta_t)\}\bar{\nu}^{1-\alpha} + (1-\hat{\epsilon})(1-\delta)\Big) \qquad (79)\end{aligned}$$

Since the level of incumbents' investment is positive, the right hand side of the first equation is increasing in $\hat{\epsilon}$. The left hand side increases as $\bar{H}$ rises. Thus, the larger the value of $\bar{H}$, the greater the invasion barrier becomes.

By the implicit function theorem, $\hat{\epsilon}$ is expressed as a continuous function of $\bar{\nu}$. If $\hat{\epsilon}$ is non-decreasing in some $\bar{\nu} < \nu_{ct}$, say $\bar{\nu}'$, then it must be the case that $\eta_t \leq 1/(1-\hat{\epsilon}(\bar{\nu}'))$, which implies that $\hat{\epsilon}(\bar{\nu}') > \bar{\epsilon}$. If $1 > (1-\bar{\epsilon})\eta_t$, (79) implies that $\hat{\epsilon}$ is increasing at ν_{ct}, which implies that there must be some $\bar{\nu}''$ such that $\hat{\epsilon}(\bar{\nu}'') = \bar{\epsilon}$ and $d\hat{\epsilon}(\bar{\nu}'')/d\bar{\nu} < 0$, a contradiction. □

Appendix J Proof of Lemma 10

Proof. Step 1

First, following the same line of argument as in the proof of lemma 9, we see $\hat{\nu} > \bar{\nu}_t$ cannot constitute a best response. Hence, we consider only mutants with $\hat{\nu} \leq \bar{\nu}_t$. There are two types of investment strategies: zero and non-zero investment strategies. We claim that, either type of mutants cannot thrive if their employment ratio $\hat{\nu}$ is greater than $\nu_{c,t}$. We take up the zero investment case first.

Case (i)-1: $\hat{\nu} \in (\nu_{c,t}, \bar{\nu}_t)$, and $\hat{\zeta} = 0$

Assume that mutants with zero investment strategy successfully enter the economy: thus, by (27), either one of the following two inequalities,

$$\left(\frac{\alpha W_t}{(1-\alpha)U_t}\right)^{1-\alpha} \bar{q}_{t+1} < (1-\delta)k_{t-1}, \text{ or } \tilde{P} < a_t^*,$$

or both of them must hold. Since $\nu_{c,t} = (1-\alpha)U_t/(\alpha W_t)$ and $\bar{q}_{t+1}/k_{t-1} = (\bar{n}_t/k_{t-1})^{1-\alpha} = \bar{\nu}_t^{1-\alpha}$, the first inequality is simplified as $(\bar{\nu}_t/\nu_{c,t})^{1-\alpha} < 1-\delta$, implying that $\bar{\nu}_t/\nu_{c,t} < (1-\delta)^{1/(1-\alpha)} < 1$, which contradicts the assumption that $\nu_{c,t} < \bar{\nu}_t$. Therefore, we must have $\tilde{P} < a_t^*$. Note also that, as shown in Figure 2, for $\hat{\nu} \in (\nu_{c,t}, \bar{\nu}_t)$, $\tilde{P} < a_t^* < MC(W_t, \hat{\nu})$, which implies that the mutants will be better off by reducing employment below $\hat{\nu}$.

Case (i)-2: $\hat{\nu} \in (\nu_{c,t}, \bar{\nu}_t)$, and $\hat{\zeta} > 0$

Suppose that, for $\nu_{c,t} < \hat{\nu} < \bar{\nu}_t$, a non-zero investment mutant strategy $(\hat{\nu}, \hat{\zeta})$ best responds to the post-entry envoronment. Note first that for $\hat{\nu} \in (\nu_{c,t}, \bar{\nu}_t)$, $MC(W_t, \hat{\nu}) > a_t^*$. (Figure 2) By Remark 4, the mutants produce less than their demand constraints, i.e., $\hat{\nu}^{1-\alpha} < \tilde{\lambda}(\epsilon, \hat{\nu})$. This implies that the mutants can increase the profits by increasing employment if the post-entry price exceeds the marginal cost, i.e., $\tilde{P}_t > MC(W_t, \hat{\nu})$. Therefore, it must be the case that

$$\tilde{P}(\epsilon, \hat{\nu}, \hat{\zeta}) = MC_t(W_t, \hat{\nu}) > a_t^*, \tag{80}$$

thus new investments must be profitable. Differentiating (38) with respect to $\hat{\nu}$ yields

$$\frac{\partial \tilde{P}(\epsilon, \hat{\nu}, \hat{\zeta})}{\partial \hat{\nu}} = \epsilon \frac{c_y W_t - \tilde{P}(1-\alpha)\hat{\nu}^{-\alpha}}{(1-\epsilon)C_R + \epsilon\{\hat{\nu}^{1-\alpha} - \max(\hat{\zeta}, 0)\}}. \tag{81}$$

Observe this derivative is negative if $\tilde{P} \geq W_t \hat{\nu}^{\alpha}/(1-\alpha) = MC_t(W_t, \hat{\nu})$. We want to show that the best responding mutants have $\hat{\zeta} < \bar{\zeta}_t$. To show this, assume on the contrary so that $\hat{\zeta} \geq \bar{\zeta}_t$. First, note that $P_t = \tilde{P}(\epsilon, \bar{\nu}_t, \bar{\zeta}_t)$ because, if the mutants play the same strategy as the incumbents, then the entry of mutants would not change output price. Next, the initial situation $(\bar{\nu}, \bar{\zeta}_t)$ is in a Keynesian temporary equilibrium, hence, $P_t > MC(W_t, \bar{\nu}_t)$. Further, by the inspection of (38), $\tilde{P}(\epsilon, \hat{\nu}, \hat{\zeta})$ is increasing in $\hat{\zeta}$, thus $\forall \epsilon$, $\tilde{P}(\epsilon, \bar{\nu}_t, \hat{\zeta}) \geq \tilde{P}(\epsilon, \bar{\nu}_t, \bar{\zeta}_t) = P_t > MC(W_t, \bar{\nu}_t)$. We know from (81) and (80) that, $\exists \delta > 0$, such that $\tilde{P}(\epsilon, \hat{\nu} + \delta, \hat{\zeta}) < MC(W_t, \hat{\nu} + \delta)$. Then, by the mean value theorem, there exists at least one point $\hat{\nu}' \in (\hat{\nu}, \bar{\nu}_t)$ such that $\tilde{P}(\epsilon, \hat{\nu}', \hat{\zeta}) = MC(W_t, \hat{\nu}')$ and $\partial \tilde{P}(\epsilon, \hat{\nu}', \hat{\zeta})/\partial \hat{\nu} > MC'(W_t, \hat{\nu}') > 0$, contradicting the above observation.

Thus, we have shown that the best responding mutants must have investment ratio $\hat{\zeta} < \bar{\zeta}_t$. Then, by Remark 5, the mutants can increase the expected profits by increasing $\hat{\zeta}$. Consequently, a mutant with $\hat{\nu} > \nu_{c,t}$ cannot survive.

Case (ii)-1: $\hat{\nu} \in (\underline{\nu}, \nu_{c,t})$

Since $a_t^* > MC(W_t, \hat{\nu})$ for $\hat{\nu}$ in this range (Figure 2), if $\tilde{P} > a_t^*$, $\tilde{P} > MC(W_t, \hat{\nu})$. Then, Remark 4 suggests that the mutants should increase employment. Therefore, we only need to consider zero investment mutants. Then, the rest of the proof is similar to that of Lemma 9. If $\tilde{P}_0(\epsilon, \hat{\nu})$ in equation (68) falls below a_t^*, zero investment becomes a best response. If $\hat{\nu} < \nu_{c,t}$ we can lower $\epsilon(\hat{\nu})$ defined by equation (72) by increasing $\hat{\nu}$. As a result, the successful mutants have strategy $(\hat{\nu}, \hat{\zeta}) = (\nu_{c,t}, 0)$. Therefore, the smallest population size of successful mutants is achieved when $\hat{\nu} = \nu_{c,t}$ in the interval $\hat{\nu} \in (\underline{\nu}, \nu_{c,t})$. Note that $a_t^* = MC(W_t, \nu_{c,t})$ so that this no-investment strategy of the mutants is also optimal employment decision.

Case (ii)-2: $\hat{\nu} \in (0, \underline{\nu})$

The proof is the same as in Lemma 9, hence omitted.

Step 2

Substituting $\hat{\nu} = \nu_{c,t}$ and $\hat{\zeta} = 0$ into (38), we have

$$\tilde{P}(\epsilon, \nu_{c,t}, 0) = \frac{-\epsilon(\bar{\nu} - \hat{\nu})c_y W_t + c_y(P_{t-1}\bar{H}/K_{t-1} + W_t\bar{\nu} + r_t P_{t-1}) + c_w P_{t-1}}{(1-\epsilon)\{\bar{\nu}^{1-\alpha} - \eta_t(\bar{\nu}^{1-\alpha} - (1-\delta)\} + \epsilon\nu_{c,t}^{1-\alpha}}. \tag{82}$$

Further, setting $\tilde{P}(\hat{\epsilon}, \nu_{c,t}, 0) = a_t^*$ by (72), invasion barrier $\hat{\epsilon}$ must satisfy that

$$- \epsilon(\bar{\nu} - \nu_{c,t})c_y W_t + c_y(P_{t-1}\bar{H}/K_{t-1} + W_t\bar{\nu} + r_t P_{t-1}) + c_w P_{t-1} \tag{83}$$

$$= a_t^* \Big[\big(\bar{\nu}^{1-\alpha} - \{\eta_t \bar{\nu}^{1-\alpha} - (1-\delta)\}\big) + \hat{\epsilon}\big\{\nu_{c,t}^{1-\alpha} - \big(\bar{\nu}_t^{1-\alpha} - \{\eta_t\bar{\nu}^{1-\alpha} - (1-\delta)\}\big)\big\}\Big] \tag{84}$$

$$= a_t^* \big[(1-\hat{\epsilon})\{\bar{\nu}^{1-\alpha} - \big(\eta_t\bar{\nu}^{1-\alpha} - (1-\delta)\big)\} + \hat{\epsilon}\nu_{c,t}\big] \tag{85}$$

As $\bar{\nu}$ increases, (83) increases while (85) decreases. Let $\Gamma(\bar{\nu}) = \nu_{c,t}^{1-\alpha} - \big[\bar{\nu}_t^{1-\alpha} - \{\eta_t\bar{\nu}^{1-\alpha} - (1-\delta)\}\big]$ in (84). Since $\eta_t\nu_{c,t}^{1-\alpha} = 1$, $\Gamma(\nu_{c,t}) = \delta$. Moreover, $\Gamma'(\bar{\nu}) > 0$. Hence, $\Gamma(\cdot) > 0$. Therefore, (84) is increasing in $\hat{\epsilon}$. Further, (83) decreases as $\hat{\epsilon}$ rises. As a consequence, $\hat{\epsilon}$ is increasing in $\bar{\nu}$. It follows that $\hat{\epsilon}$ is increasing in $\bar{H}$ because (83) is increasing in $\bar{H}$. □

Appendix K Proof of Proposition 4

Proof. First, let us obtain invasion barrier for $\bar{\nu} = \nu_{ss}$. It follows from (44) and (46) that the left hand side of (78), denoted by C_N, is expressed as $C_N = P_{ss}c'$, where $c' = c_w/(1 - c_y)$. Also, by (43) and (44), we get $\eta_{ss} = K_{ss}/Q_{ss} = 1/(c' + \delta)$ and $\nu_{ss}^{1-\alpha} = c' + \delta$ respectively. Therefore, arranging terms in (78), we get

$$\begin{aligned}\hat{\epsilon} &= 1 - \frac{a_{ss}\nu_{ss}^{1-\alpha} - P_{ss}c'}{\delta a_{ss}} \\ &= 1 - \frac{c' + \delta}{\delta} + \frac{P_{ss}}{a_{ss}}\frac{c'}{\delta} \\ &= \frac{c'}{\delta}\left(\frac{P_{ss}}{a_{ss}} - 1\right) \\ &= \frac{c'}{\delta}(c' + \delta)^{\frac{1}{1-\alpha}}\bar{H}, \end{aligned} \tag{86}$$

where the last equality follows from (52). Since $\eta_{ss} = 1/(c'+\delta)$, the condition, $1 > (1 - \bar{\epsilon})\eta_t$, in lemma 9 is expressed as

$$\hat{\epsilon} < 1 - c' - \delta. \tag{87}$$

Combining (86) with (87), we see that, if $\bar{H}\{c_w/(1 - c_y)\}^{1/(1-\alpha)} < \delta\{(1 - \delta)(1 - c_y)/c_w - 1\}$ at $\bar{\nu} = \nu_{ss}$, $\hat{\epsilon}$ is decreasing function of $\bar{\nu}$. Hence, the condition of lemma 9 holds for all $\bar{\nu} \leq \nu_{ss}$, as desired. □

Appendix L Appendix for Referee

Derivation of MC_t in (20)

From (19), we get

$$\begin{aligned} MC_t &= \frac{W}{1 - \alpha}\left(\frac{n_t}{k_{t-1}}\right)^{\alpha} \\ &= \frac{W}{(1 - \alpha)}(k_{t-1}^{\alpha-1}n^{1-\alpha})^{\frac{\alpha}{1-\alpha}} \\ &= \frac{W}{(1 - \alpha)}\left(\frac{\bar{q}_t}{k_{t-1}}\right)^{\frac{\alpha}{1-\alpha}} \\ &= \frac{W}{(1 - \alpha)}\lambda_t^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Derivation of Equation (38)

$$\tilde{P}(\epsilon,\hat{\nu},\hat{\zeta}) = \frac{c_y\big(P_{t-1}\bar{H}N_f + W_t\tilde{N}_t(\epsilon,\hat{\nu}) + r_tP_{t-1}K_{t-1}\big) + c_w(P_{t-1}K_{t-1})}{\tilde{Q}_t(\epsilon,\hat{\nu}) - \tilde{Z}_t(\epsilon,\hat{\zeta})} \tag{37'}$$

$$= \frac{c_y\{P_{t-1}\bar{H}N_f + W_t\big((1-\epsilon)N_t + \epsilon\hat{N}\big) + r_tP_{t-1}K_{t-1}\} + c_w(P_{t-1}K_{t-1})}{(1-\epsilon)\bar{Q}_t + \epsilon K^\alpha\hat{N}^{1-\alpha} - (1-\epsilon)(\eta_t\bar{Q}_t - (1-\delta)K) - \epsilon\max(\hat{Z},0)}$$

$$= \frac{c_y\{P_{t-1}\bar{H}N_f + W_tN_t + r_tP_{t-1}K_{t-1}\} + c_w(P_{t-1}K_{t-1}) - c_yW_t\epsilon(N_t-\hat{N})}{(1-\epsilon)\big(\bar{Q}_t - \eta_t\bar{Q}_t + (1-\delta)K\big) + \epsilon\big(K^\alpha\hat{N}^{1-\alpha} - \max(\hat{Z},0)\big)}$$

$$= \frac{C_N - \epsilon(\bar{\nu}-\hat{\nu})c_yW}{(1-\epsilon)C_R + \epsilon\big\{\hat{\nu}^{1-\alpha} - \max(\hat{\zeta},0)\big\}}. \tag{38'}$$

Derivation of Equation (49)

$$c'(c'+\delta)^{-1/(1-\alpha)} = \bar{H} + \frac{(1-\alpha)(r+\delta)}{\alpha}(c'+\delta)^{-1/(1-\alpha)} + r(c'+\delta)^{-1/(1-\alpha)}$$

$$c' = \bar{H}(c'+\delta)^{1/(1-\alpha)} + \frac{(1-\alpha)(r+\delta)}{\alpha} + r$$

$$c' + \delta - \bar{H}(c'+\delta)^{-1/(1-\alpha)} = \frac{r+\delta}{\alpha}$$

$$(c'+\delta)\{1 - \bar{H}(c'+\delta)^{\alpha/(1-\alpha)}\} = \frac{r+\delta}{\alpha} \tag{49'}$$

Derivation of (81)

Let $\Psi = (1-\epsilon)C_R + \epsilon\big\{\hat{\nu}^{1-\alpha} - \max(\hat{\zeta},0)\big\}$. Differentiating (38) with respect to $\hat{\nu}$, we obtain

$$\frac{\partial\tilde{P}(\epsilon,\hat{\nu},\hat{\zeta})}{\partial\hat{\nu}} = \frac{\epsilon c_yW\Psi - \big(C_N - \epsilon(\bar{\nu}-\hat{\nu})c_yW\big)\epsilon(1-\alpha)\hat{\nu}^{-\alpha}}{\Psi^2}$$

$$= \frac{\epsilon c_yW\Psi - \tilde{P}\Psi\epsilon(1-\alpha)\hat{\nu}^{-\alpha}}{\Psi^2}$$

$$= \epsilon\frac{c_yW_t - \tilde{P}(1-\alpha)\hat{\nu}^{-\alpha}}{(1-\epsilon)C_R + \epsilon\big\{\hat{\nu}^{1-\alpha} - \max(\hat{\zeta},0)\big\}}. \tag{81'}$$

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