Investment Dynamics, Profitability and Business Cycle

Ichiro Takahashi and Isamu Okada*

February 19, 2011

Abstract

We attempt to construct a simple agent-based model of real and financial sectors that can reproduce business cycles. The firms in this model takes into account the relative price between investment and consumption goods as well as the demand constraint in deciding how much to invest, and how many workers to employ. Our artificial macro-economy reproduces business cycles with some properties that are consistent with some salient facts. Two types of economy, high-performing and low-performing economies, emerge with different degrees of downward rigidity of caital price. In a low-performing economy, low profitability discourages investment, which in turn causes the low level of investment.

Keywords: Business Cycle, Unemployment, Agent-based Simulation

1 Introduction

This paper provides an agent-based model that focuses on the dynamic interaction of investment activities among firms. In determining its level of investment a firm takes into account the following two factors: the level of demand for its product which is partly influenced by how actively other firms

*We are thankful to Takeshi Ojima for helpful comments. This research is supported by Grants-in Aid 22530284 from the Ministry of Education, Culture, Sport, Science and Technology of Japan.

are investing, and the supply price of investment goods which also depends on investment levels of other firms. To capture these features, this paper constructs an agent-based model of business cycle having the following features:

(1) An individual firm, taking the macro-level demand constraint and other aggregate variables as exogenously given, decides how much to invest and how many workers to employ. These decisions in turn collectively determine the aggregate demand in the subsequent periods;

(2) In each period only a fraction of firms are given investment opportunities and they take time to build their production facilities.

(3) In contrast to neoclassical macroeconomic model, agents in this model are myopic and follow heuristic behavioral rules.

(4) All the prices adjust slowly, e.g., the commodity price, the money wage rates and the interest rate.

(5) the expansion and contraction of credit money are determined endogenously through banking system in response to investment and repayment by firms.

The first feature is the heart of Keynesian economics. An agent-based approach allows us to incorporate this constraint naturally. (See for example, Gatti et al. (2008) In neoclassical macroeconomics, a single firm typically represents the entire production sector. The construction enables the firm to relax the aggregate demand constraint by unilaterally increasing its investment and employment. The second feature implies that there is a time lag between the expansion of aggregate demand caused by investment spending and the resultant expansion of production capacity. This lag plays an important role in generating business cycle.

Our main results obtained by the simulations are as follows: (1) Our small artificial economy reproduces business cycles under a wide range of parameter sets, wherein the simulated behavior is fairly consistent with real data. (2) Two types of economy can emerge: high-performing and low-performing ones.

The second point is related to Minsky's insight about profitability and aggregate investment. Minsky (1986) shows that "in the simplest formal case the proximate determinants of how our economy works are the subsystems that determine (1) money wages, (2) the average productivity of labor, and (3) the ratio of investment employment to consumption employment." The novelty of this paper is that it endogenizes the third factor and incorporates it into investment decision in a coherent way: Investment decisions at micro level are based on profitability, which collectively determines the rtio of

investment employment to consumption employment. Conversely, the ratio determines the overall profitability for individual decision makers. The downward rigidity of capital price turns out to be the main determinant of how well an economy performs.

Closely related to this paper are Gatti et. al. (2003), Napoletano,(2005) and Gaffeoa et. al.(2008). Endogenous heterogeneities included in these models such as technological progress and commodity prices do not appear in this paper. However, our model incorporates fixed capital formation with endogenous investment goods price that do not appear in their models.

Section 2 describes th model. Section 3 presents results and discuss them. Section 4 cocules.

2 Model

2.1 Overview

This subsection explains the structure and timeline of the model. There are five markets in the economy: financial market, consumption goods market, capital goods market, and two labor markets for consumption goods and capital goods. All the prices in these markets adjust slowly. The economy consists of a large number of firms producing consumer goods (called hereafter firms), an investment goods producer (called the company), a single bank, a shareholder, two types of workers employed by the two industries (called workers and carpenters respectively). There is only credit money (saving accounts) in this economy. Time is discrete and has two layers: month and year with one year being M months. Investment period is one year and production and employment period is one month. It takes M months (gestation period) to build production facility (called factory), which can be used for N years after its installation. Two types of workers are separated and not allowed to change their types.

In each period the agents in this model routinely make investment, consumption and investment decisions. The timeline of the model is as follows:

(T1) The prices of consumption goods, factory, and the money wage rates, and the interest rate are announced to all the agents.

(T2) Each agent receives interest income and/or makes interest payment from and to the bank based on her amounts of saving and loan outstanding. The firms with loan outstanding pays back a part of principal payment.

(T3) Each firm pays dividend that was saved from profits from the previous period to the shareholder, which collectively determines the dividend income of the shareholder. Together with (T2) the income of the shareholder is determined.

(T4) The firms with investment options place an order of capital goods to the company. (They make payment to the builder M months later upon their installation with the fund financed by the bank.)

(T5) The price of capital goods is determined. The company starts producing capital goods by hiring carpenters upon receiving the orders from the firms. It continues to hire them for M months consecutively (including this month) until the construction is completed, which gives wage income to the carpenters. The payment is also financed by the loan from the bank.

(T6) For given capital stock, all the firms including those with the investment choices decides how many workers to employ and start producing consumption goods. This, in turn, determines the wage income of the workers.

(T7) All the households decides on their consumption level, hence on the amounts of saving.

(T8) The firms complete production for this period.

(T9) The aggregate demand for consumption goods is distributed among firms in proportion to its production capacity. Any gap between production and demand is filled or absorbed by inventory changes.

(T10) All the prices are revised. The interest rate is updated to reflect the inflation rate and the lender's risk, which in turn depends on the relative amount of the total loan outstanding to the bank's equity. The consumption goods price and the wage rates adjust to the corresponding excess demand or supply. The capital goods price is determined automatically by the wage rate of carpenters and the interest rate.

(T11) Balance sheet items of all agents are revised.

The next subsection provides assumptions related to investment decision and explains how a firm in this model selects the level of investment which is the fuel that drives this economy.

2.2 Firm Producing Consumption Goods

Based on the point estimate of the demand for its product, $\hat{x}_t^i$, the expected price of capital goods, $\hat{v}_t$, and price information (the price of consumer goods

p_t, the money wage rate of workers w_t, and the rate of interest r_t), a firm with investmen option (call it Firm i) determines the level of investment I_t^{i1}. At the beginning of each month, each firm also chooses its amount of employment L_t^i and the quantity of output based on only price information and not on quantity constraint[2] Each firm finishes production at the end of period and learns the realized sales.

Each firm has the following linearly homogeneous Cobb-Douglass production function:

$$q_t^i = A(K_t^i)^\alpha (L_t^i)^{1-\alpha} \text{ for all } i = 1, 2, \dots J, \tag{1}$$

where q, A, K and L denote the quantity of output, the level of technology and the amount of capital stock and employment.

The firms in this model are homogeneous both in size and balance sheet. The shareholder provides E_0^i dollars of initial capital to each firm, thus creating J firms in total. (J is the natural number multiplication of M.) To start the business the firms purchase factory from the shareholder by borrowing from the bank. We assume that, once creating a firm, the shareholder does not increase capital so that investment fund is entirely financed by the bank. Moreover, for transparency, the firms are not allowed to use their deposits.)

Let K_t^i, D_{t-1}^i and B_{t-1}^i denote the number of factories, the amount of deposit and loan outstanding that Firm i possesses or owes, respectively at the beginning of period t ($j = 1, 2, \dots J$). Since the firm repays a part of loan principal that exactly matches the depreciation cost of factory every period, the book value of the remaining capital always balances with the amount of loan outstanding.

Now we examine how a firm makes investment decision. Investment decision is central because it constitutes aggregate demand as well as future production capacity. Refer to Appendix for the derivation.

[1]In this model, for expositional ease, firms take turns in having option to invest. Alternatively, options are given to those firms that are randomly selected. In either specification, the main results remain unchanged.

[2]We neglect demand constraint in monthly decision-making on employment. This assumption is acceptable if the length of one period is short enough since a firm can accommodate excess supply or demand by changes in inventory. Our model uses sales performance in the previous period as the point estimate in forecasting sales. If forecast error is to be taken into account, an optimal output policy should allow for occasional over production. However, point estimate forces firms to have no inventory, hence decreasing output continuously until the whole economy collapses. A more detailed discussion is given below.

The expected unit cost of consumption goods AC is given by

$$AC_t = \frac{1}{A}\left(\frac{\hat{v}_t}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}.$$

The linear homogeneity of the production function suggests the desirable level of capital stock K^* is given by

$$K_t^* = \begin{cases} \frac{1}{A}\left(\frac{\alpha w_t}{(1-\alpha)\hat{v}_t}\right)^{1-\alpha} & \text{if } p_t \geq AC_t \\ K_{t+M} & \text{otherwise,} \end{cases}, \tag{2}$$

where K_{t+M} is the size of factory available at the beginning of period $t+M$ provided that the firm postpones its investment.

Thus, the firm has the following investment rule:

$$I_t^i = \begin{cases} \min(K_t^{*i} - K_{t+M}^i, \bar{I}) & \text{if } Eq_t^i = \frac{B_t^i}{B_t^i + D_t^i} \geq \gamma \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where $\bar{I}$, Eq_t^i, and γ denote the maximum size of new investment a firm can make, and the minimum capital-asset ratio required by the lender, respectively.

2.3 Parameter Setting

The following set of parameters are used. We will check more detailed sensitive analysis in the future.

p_0	120	$\hat{v}_0$	120	w_0	0.5	r_0	0.05
A	1.25	α	0.3	E_0	$3e4$	J	10
M	10	N	10	K_0^i	10	γ	5.0
θ_D	0.1	θ_π	0.1	$\bar{c}$	4	c_f	0.85
c_s	0.04	$\bar{c}^K$	4	c_f^K	0.85	c_s^K	0.04
$\bar{c}^S$	0	c_f^S	0.55	c_s^S	0.01	ϕ	10
θ_1	0.01	θ_r	$1e-4$	θ_w^U	0.03	θ_w^D	0.03
$\theta_{w^K}^U$	0.03	$\theta_{w^K}^D$	0.03	θ_w^K	1.0		

3 Simulation Results and Discussion

Our main simulation results are the following: (1) Our small artificial economy robustly reproduces business cycles, wherein the simulated behavior of

the economy is fairly consistent with observed facts. (2) Two types of economy can emerge: high-performing economy and low-performing ones[3]. The Downward rigidity of carpenter's wage rate θ^D_{wK} is decisive in determining into which type an economy evolves.

3.1 Business Cycle

FIrst, we show that business cycles are generated under a standard scenario. Then, we explain the basic mechanism of the fluctuations. In the standard scenario where $\theta^U_w = \theta^D_w = \theta^U_{wK} = \theta^D_{wK} = 0.03$ all balance- sheet items repeat cycles regularly. (Figure 1) These cycles are driven by investment cycle: the economy fluctuates with alternating periods of high- and low-intensity of investment activity. (Figure 2) The result is also consistent with some salient facts: both money supply (Figure 3) and the real wage rates (Figure 4) move procyclically and new construction is a leading indicator while the interest rate is a lagging indicator (Figure 5).

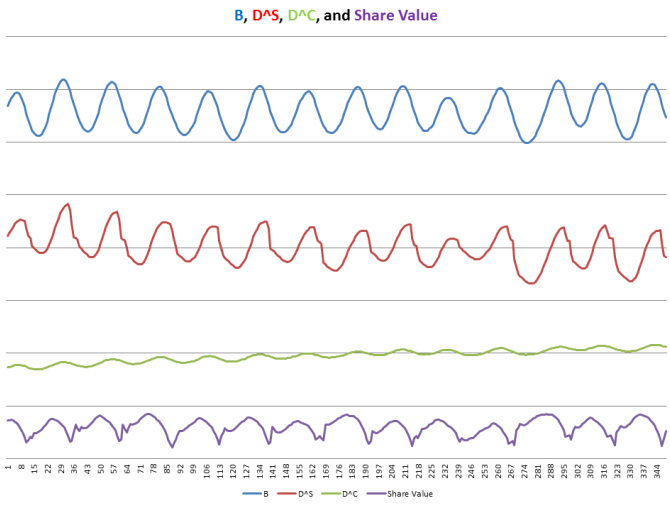

Figure 1: Simulation results for the standard scenario

[3]For more rigorous results, a formal model should be constructed. Takahashi (2011) ("Good and Bad Business Cycles" mimeo) confirms the main results obtained hold in a simplified model.

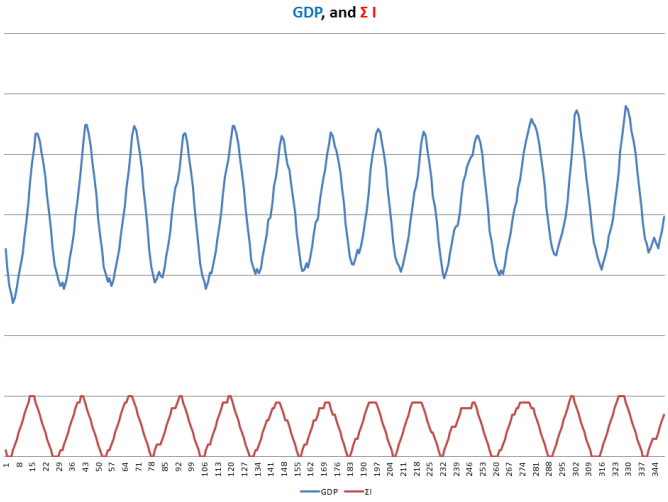

Figure 2: GDP and Investment

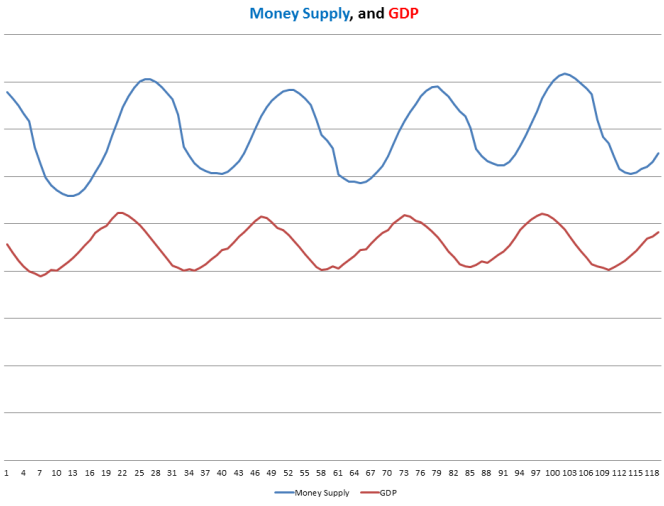

Figure 3: Money Supply and GDP

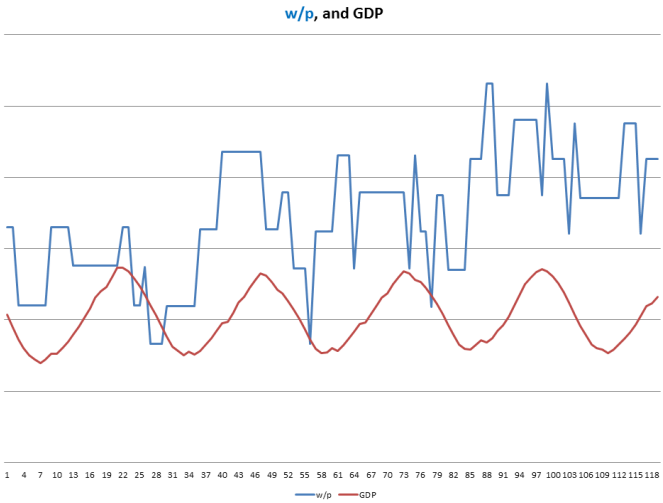

Figure 4: Real Wage and GDP

The cycles arise because investment generates positive feedback loop through aggregate demand and there are upper- and lower-bounds in aggregate investment[4]. Consider first the positive feedback of investment.

The existence of the positive loop is obvious because aggregate investment is a part of X_{t-1}, which appears in equation (2) through x_{t-1}. Suppose the the economy recovers from a long recession. An increase in investment stimulates the aggregate demand, which increases the next period investment by equation (2). Until the unemployment rate of carpenter reaches the natural unemployment rate, w_t^K continues to decline, which also stimulates investment through falling price of capital $\hat{v}_t$ by equations (**??**) and (**??**) as shown in Figure 5.

The above destabilizing nature of investment is similar to that of acceleration principle model analyzed by Samuelson (1939) and Hicks (1950). What is unique in this model is the existence of time lag in expanding and reducing the production capacity. In a conventional model with a gestation period of one, the positive feedback effect of investment is partially canceled out by increasing production capacity. By contrast, in this model, it takes M periods for a new factory to start production. Thus, new construction can stimulate aggregate demand without increasing the supply of the consumption goods,

[4]As will be seen below, they are endogenous.

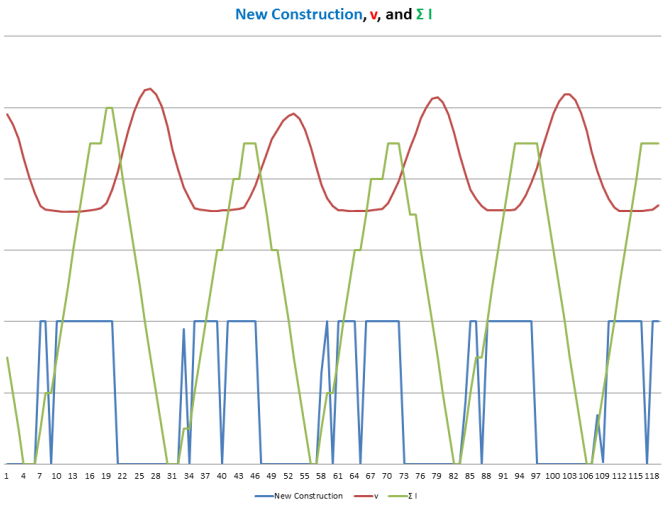

Figure 5: New Construction, capital Price and Investment

which raises the relative price of consumption goods, helping maintain favorable market conditions for investment. This gives rise to instability. Figure 6 shows that as new investment accumulates, the price of factory, the wage rate as well as the rate of interest continue to rise (these effects are summarized by K^*), and the aggregate production capacity (denoted by K) does not expands for several periods in a recovery phase. The instability also arises in recession. When new construction of factory comes to a halt, it takes M periods further until production capacity ceases to grow because projects already under construction cannot be stopped immediately. This creates undesirable over-production.

Is there a ceiling or floor to these fluctuations? What determine their levels? How does the investment dynamics hit a ceiling and floor? According to Hicks (1950) full employment output forms the ceiling whereas autonomous spending gives the bottom. The ceiling and the floor are also present in our model. However, the fluctuations in this model is more complex. Before explaining the differences that exist between the ceiling and floor of Hicks' model and ours, we need to explain our mechanism of fluctuation.

The ratio $\frac{w_t}{\hat{v}_t}$ in equation (**??**) plays the central role in our investment dynamics. Since w_t is far more stable than w_t^K [5], and r_t and w_t^K move in a

[5]This is because consumption demand is more stable because of the autonomous spend-

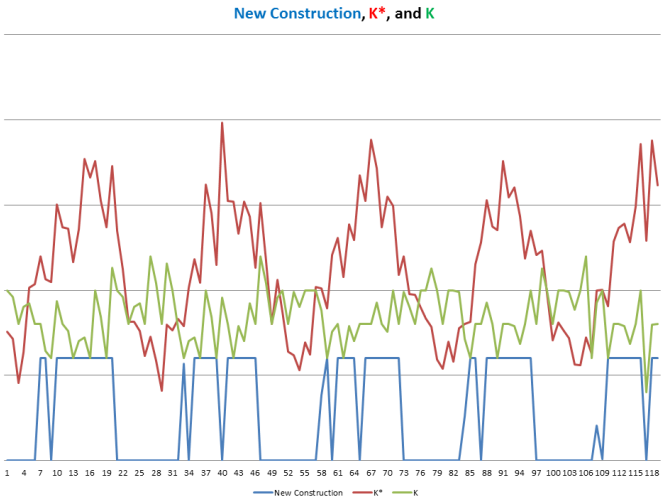

Figure 6: New Construction, K^* and K

parallel fashion, we focus on the carpenter's wage as a proxy of capital price. As investment continues to expand, the unemployment rate of carpenter gradually decreases below the natural unemployment rate. This leads the carpenter's wage rate to rise, which discourages investment by lowering the desirable capital-labor ratio k^*. This negative feedback through w_t^K and r_t is reinforced by expanded production capacity induced by investment. These factors undermine favorable investment conditions, thus causing recession. In short, an economic expansion will inevitably hit the ceiling because the capital price becomes relatively (to worker's wage rate) too costly. In a similar fashon, The economy starts to pick up after a long recession: Figure 5 shows that even when the aggregate demand is still relatively low, a firm finds it profitable to resume investment as the factory price falls sufficiently low.

Now we are ready to investigate the endogeneity of the ceiling and floor to fluctuations. The subsequent discussion may shed some light on understanding the existence and emerging of low-performing economy.

ing and wealth effect.

3.2 Business Cycle with Unemployment

Compare performances of two simulations generated with different parameter sets. Figure 7 shows GDP of the standard scenario and "downward rigidity" scenario (referred as DWR) where $\theta_w^U = \theta_w^D = \theta_{wK}^U = \theta_{wK}^D = 0.003$. The economic performance of the latter scenario is much worse than the standard scenario. This result appears to be general. Figure 8 indicates that the more downwardly rigid the nominal wage of carpenter becomes, the smaller and the more volatile GDP becomes.

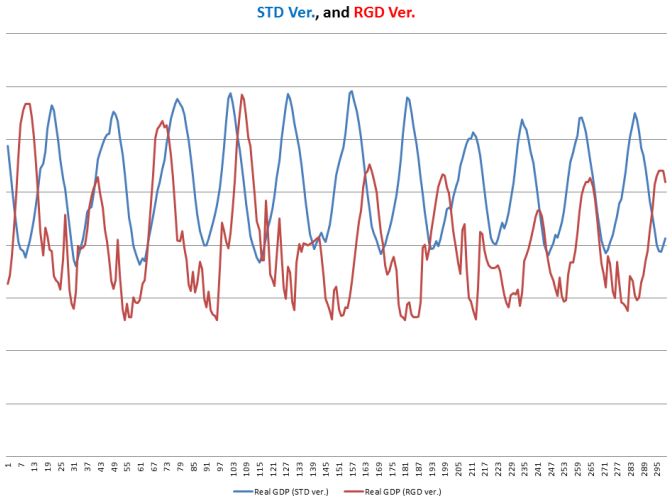

Figure 7: GDP under Standard and DWR scenarios

Figure 9 and 10 provide the clue to understand the essential cause of their difference. Figure 9 shows that, in the standard scenario, the price of consumption goods is away above the average cost. By contrast, in DWR scenario (Figure 10), they are intertwining with each other. In other words, the profit condition in equation (2) kicks in the low-performing economy. This additional constraint in investment decision knocks an investment plan. As a result both the quantity of factory K and financial asset D are significantly smaller than those of the base scenario. The determinant of profitability measured by $\frac{p_t}{AC_t}$ is the ratio of carpenter's wage income to worker's wage income. (See Chapter 7 of Minsky (1986)) The low level of investment in DWR scenario causes its low profitability. Conversely, the resultant low profitability causes the low level of investment.

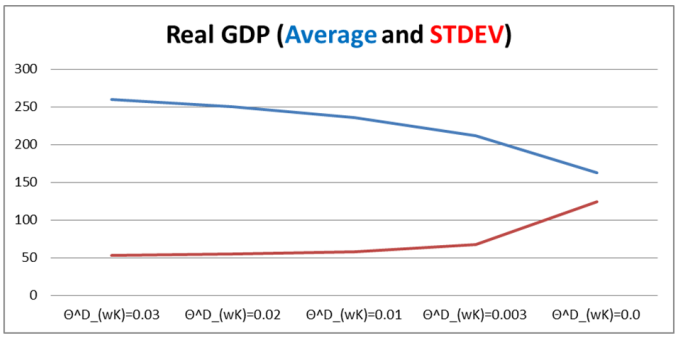

Figure 8: Average and Standard Deviation of GDP and DWR

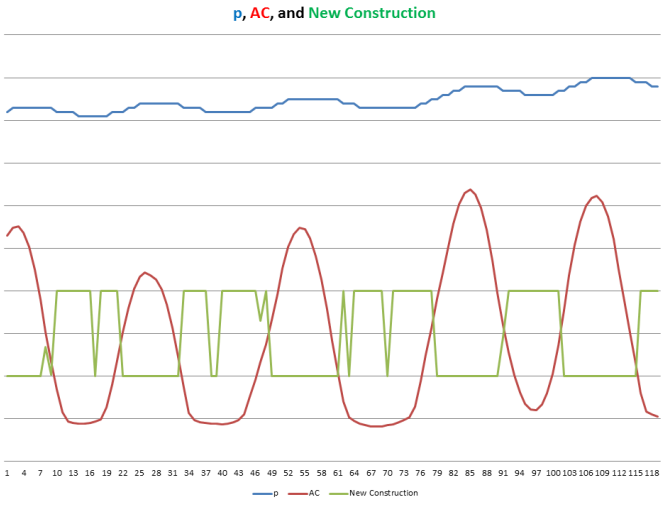

Figure 9: Price and AC under Standard Scenario

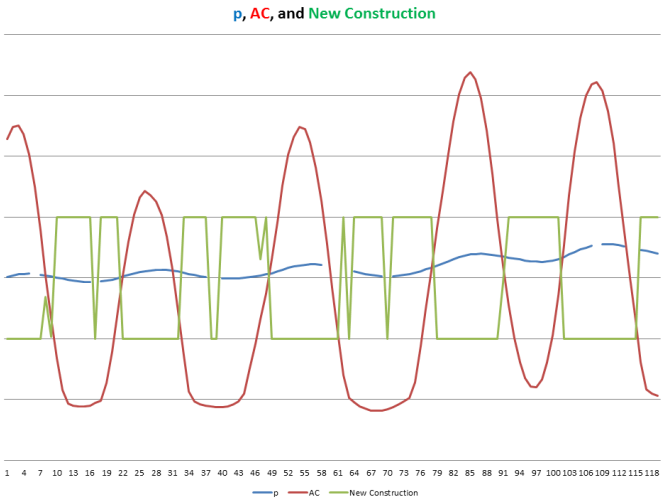

Figure 10: Price and AC under Downward Rigidity Scenario

The above discussion suggests that while "physical restriction of full employment" can form the highest possible ceiling but it is not the only one. As Figure 7 shows, it appears that a low-performing economy has a ceiling located below that of high-performer. In a low-performing economy, due to the low wage of workers, rising wage of carpenters makes their investment projects unprofitable before the economy hits the physical constraint of full employment. Moreover, the low real wage of workers is a consequence of low capital-labor ratio. In other words, the level of ceiling is not exogenous but determined endogenously.

Similarly, the bottom of fluctuations is also an endogenous variable. It is the sum of autonomous spending and real balance and interest income effects that offsets the drops. The low level of investment pushes down the bottom by decreasing K hence B.

The natural question is why the economy gets stuck with the unemployment economy. With the downward rigidity of carpenter's wage impedes the recovery of profitability, discouraging investment. This implies the low profitability and thus resulting in the low level of investment. Because the quantity of capital stock in our model is the real financial wealth, low level of investment can be interpreted as the result of shortage of consumption demand due to the poor wealth.

Once economic downturn begins, both production capacity and aggregate consumption demand decrease. In order to restart investment, the aggregate demand should reduce at a slower pace than the production capacity does. In determining the relative speed the amount of real wealth is crucial. It is important to recover profitability while the wealth is intact so that the aggregate demand is still strong[6] Otherwise, the resultant low level of investment causes low profitability in addition to demand constraint, which discourages investment thus induce perpetual unemployment.

4 Concluding Remarks

To focus on non-financial aspect of the investment dynamics, we constructed a model which is essentially that of one real sector. The simulation results shed some light on understanding the nature of business cycle and the cause of unemployment. If the steady state equilibrium is stable, we can identify what determines economic performance by examining it. Since it is not the case, we need to ask what factors determine the average performance of the economy.

The simulated behaviors exhibited perpetual business cycles. Moreover, they were largely consistent with some observed facts. We also found that the performance of the economy varies significantly with the values of parameters. In particular, the degree of downward rigidity of capital price plays a decisive role in determining the performance. With its strong downward rigidity, the economy fluctuated around the low level of investment hence GDP.

In order to keep healthy economy we came to a conclusion that it is important to regain profitability in investment at an earliest stage of recession. This implies that it is vital for an economy to sustain the financial wealth of households. For this reason, monetary authority should avoid bubble forming.

Many directions for extension are possible. The investment industry should also engage in investment. A central bank needs to be modeled to allow for cash and researves. Furthermore, to address financial instability, the demand price of factory and stock market should be modeled.

[6]The autonomous spending supports the economy. However, without the help of real wealth effect the autonomous spending alone is too small to achieve full employment as shown in DWR.

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