The Choice of Wage Policy and Dualism in the Labor Market

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Abstract

The purpose of this paper is to characterize an equilibrium in which some firms post a wage, while others bargain with workers. We show that such an equilibrium exists under a situation where workers are homogeneous and employed workers engage in on-the-job search. Our model provides a simple theoretical framework for studying the labor market dualism that involves various issues of disparities between regular and non-regular workers. This paper shows that the proportion of firms adopting wage bargaining is lower than the socially optimal level for a decentralized equilibrium. Furthermore, we find ways to increase this proportion and to improve social efficiency.

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1 Introduction

The purpose of this paper is to characterize an equilibrium in which some firms post a wage, and others bargain with workers, focusing on changes in the composition of these firms. Both posting a wage and wage bargaining are major wage setting policies that are

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frequently used in models of job search. [1] Unlike many past researchers, we do not fix one of two wage setting mechanisms. Rather, firms choose the optimal one when they open a job vacancy. If firms choose negotiation, job seekers and firms who meet in the search process make a match and determine a wage level under the given bargaining power of each agent and market conditions. In the standard model (e.g., Mortensen and Pissarides (1994), and Pissarides (2000)), a bargained wage is characterized by a solution to the Nash bargaining problem. On the other hand, if firms post a wage, they determine a wage offer that maximizes their expected profits and announce it to the market. When they succeed in making a match, they pay this predetermined wage to the selected worker (because of reputation effects). In this paper, we identify conditions for an equilibrium such that firms adopting different wage policies coexist and examine properties of the equilibrium.

Wang (1995), Arnold and Lippman (1998) and Camera and Delacroix (2001)(2004) investigate how a seller of a single good chooses a selling price mechanism (price posting or bargaining). In these studies, buyers are assumed to be heterogeneous and to have some private information such as their evaluation to the good. The main purpose of these studies is to identify the situation in which a seller adopts each price setting mechanism.

Ellingsen and Rosen (2003) and Michelacchi and Suarez (2006) extend the above models to investigate the labor market. By using the random matching framework, Ellingsen and Rosen (2003) focus on the situation in which all firms choose to bargain with a worker in equilibrium. They suppose that workers' productivity is heterogeneous and is perfect information. They show that firms prefer wage negotiation to posting a wage as the distribution of workers' productivity becomes more dispersed. The same result is obtained in Michelacchi and Suarez (2006), whose model is based on the directed search approach. [2] They assume that workers' productivity is private information and indicate that a combination of this assumption and the directed search structure yields the adverse selection problem as well as the search inefficiency due to the search externality. Note that Michelacchi and Suarez specify the critical values of a worker's bargaining power that make up each wage setting mechanism as an equilibrium regime: a pure posting regime, a pure bargaining regime and a mixed regime.

One of our objectives is to examine how the choice of a firm's wage policy depends on the market environment by using a random matching model. Ellingsen and Rosen (2003) show that the equilibrium in which all vacant firms bargain does not exist when the dispersion of worker productivity is very small. In contrast, we construct a model with homogenous workers, but incorporate on-the-job search into the model. [3] We can then characterize an

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1) Pissarides (2000) develops various models based on the equilibrium search model with bilateral bargaining. In contrast, Burdett and Mortensen (1998) established a wage-posting model and characterized a continuous wage offer distribution. Mortensen (2003) is a comprehensive work dealing with the latter approach.

2) In the random matching approach, job seekers and job vacancies meet randomly during search (see Mortensen and Pissarides (1994) and Pissarides (1994)(2000)). In the directed search approach, on the other hand, job seekers are permitted to make free applications to favorable jobs (Moen (1997) and Acemoglu and Shimer (1999)).

3) In Pissarides (1994), employed workers accumulate firm-specific human capital, and this accumulation results in wage increases as workers stay longer in their current jobs. There are two types of jobs (high productivity jobs and low productivity jobs), and only workers in the low productivity jobs engage in job search. Although human capital accumulation is not introduced in our model, the structure of on-the-job

equilibrium in which some firms choose wage posting and others choose wage bargaining under some values of parameters. Ellingsen and Rosen (2003) do not focus on the emergence and properties of this type of equilibrium. We have a particular interest in this equilibrium, since the labor market in Japan has recently become dualistic; some workers receive high wages and high employment protection, including indefinite employment periods, while others have low wages and weak protection, including fixed-term contracts (as is indicated in OECD Economic Survey Japan (2006)(2008)). According to the literatures such as Cousins (1999), Yeandle (1999), Gustafsson at al.(2003), Hoffman and Walwei (2003), and Osawa and Houseman (2003), this phenomenon is not specific to Japanese economy and but has been observed in U.S. and many European countries. This paper provides theoretical solutions to resolve this type of dualism. [4] There are few theoretical studies that deal with this dualism while many studies, as stated above, investigate the current position of non-regular workers in a number of countries. The main purpose of this paper is, therefore, to construct a theoretical model that is useful to study an ideal form of the labor market with a dualistic structure.

The disparity in wages, fringe benefits, and employment stability between regular workers and non-regular workers is one of the 'hot' issues in the Japanese economy. Other countries such as U.S. and U.K. confront a similar problem (see Gustafsson et al. (2003) and Houseman and Osawa (2003)) while in Netherlands, Sweden and Denmark, institutions that force employers to treat their employees equally irrespective of employment status are considerably in place compared to other European countries (see Cousins (1999), Yeandle (1999), Gustafsson et al.(2003) and Hoffman and Walwei (2003)). It is difficult to strictly classify working people into regular and non-regular groups because the definitions of these groups differ between countries. [5] Even so, we can point out several important features necessary to describe non-regular jobs: (i) non-regular workers face shorter employment periods than regular workers; (ii) regular workers generally perform more complex and important tasks than non-regular workers. The first feature indicates that employing non-regular workers allows firms to easily adjust employment under uncertain demand conditions. As stated in Rebick (2005) and OECD (2008), this is one of the main reasons why firms hire non-regular workers. The second feature justifies the discrimination between regular and non-regular workers. Honda (2001) and Shimizu (2007), however, show that the differences in assigned tasks between these groups of workers have recently become ambiguous. [6] This is related to firms' objective to reduce labor costs. In an OECD report

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search developed in Pissarides (1994) is useful for our study.

4) In Acemoglu (2001), there are two types of jobs (jobs with high productivity and jobs with low productivity) in the economy, and the composition of these jobs is determined in equilibrium. Unlike our model, wages are given as a solution of the Nash bargaining problem in each job, and on-the-job search is not introduced to the model. From another point of view, our model corresponds to the case in which employed workers in bad jobs have no bargaining power.

5) See Suzuki (1998) and Ogura (2002) for an international comparison of classifications of employment status. They indicate that, in general, countries with strong employment protection systems have a high proportion of non-regular workers. On the other hand, countries in which even regular workers face a high turnover rate do not always experience an increase in the number of non-regular workers.

6) In the 2007 Economic Survey of Japan reported by the Cabinet office, a substantial number of business establishments acknowledged the presence of part-time workers who engaged in the same tasks as regular workers. Furthermore, these establishments also responded that these part-time workers and regular

(2008), this is the most important reason for hiring non-regular workers. We show in our model that workers hired into non-regular jobs obtain their reservation wage and examine how the composition of jobs and the wage differentials respond to a change in parameters characteristic of non-regular type jobs.

According to the data provided by the Bureau of Statistics in the MIC (Ministry of Internal Affairs and Communications), the number of non-regular workers has increased steadily, while the number of regular workers turns to increase in 2006. The proportion of non-regular workers has reached 30%. Furthermore, we must pay attention to the fact that the increase in the number of regular workers is offset by the increase in the number of non-regular workers (In many European countries, a large part of new jobs has been created in the form of non-regular jobs recently. See Cousins (1999) for example). As a result, the ratio of regular workers to non-regular workers is decreasing year by year. We regard jobs with a posted wage (a negotiated wage) as non-regular (regular) ones. In this paper, we characterize an equilibrium mixture of these different types of jobs and determine a policy that makes the relative ratio of regular workers increase. Since Ellingsen and Rosen (2003) do not consider this type of equilibrium, their model could not describe the existence of jobs which adopt diverse wage policies. [7)]

With these motivations, we obtain the following results: (i) even when workers and firms are homogeneous, there exists an equilibrium in which some firms adopt posting wages and others adopt wage negotiation, by assuming on-the-job search. (ii) In this equilibrium, there exists an excess supply of jobs that adopt posting wages, and this reduces social surplus below its efficient level. (iii) Smaller differences in job productivity make firms reluctant to choose bargaining. This implies that the number of workers hired at firms corresponding to regular jobs declines. (iv) As workers in jobs with a posted wage are fired more frequently, more firms choose bargaining and the ratio of workers in these firms increases. Furthermore, in this case, the wage gap between jobs becomes larger. (v) An introduction of a minimum wage greater than the posted wage has the same impacts on the choice of a wage policy and on the ratio of regular workers as does the rise in the firing frequency specific to non-regular jobs. However, its impact on the wage gap is ambiguous.

This paper is organized as follows. In Section 2, we explain the basic framework of our model, including the matching technology. In Section 3, we characterize a wage determined by each type of job. In Section 4, job creation conditions are derived. In Section 5, we show the unique existence of an equilibrium in which some firms adopt posting wages and others adopt wage negotiation under some parameter values. In Section 6, social efficiency and properties of the equilibrium are investigated. In Section 7, our conclusions are provided.

workers are treated differently in terms of wage compensation. Therefore, it is desirable for firms to hire less costly part-time workers rather than regular workers.

7) The existence of different types of jobs and the occurrence of wage differentials are represented by the model of dual labor markets, such as in Saint-Paul (1996). This market is composed of a primary sector where monitoring is imperfect and efficiency wages are paid and a secondary sector where monitoring is perfect and competitive wages are paid. Firms require this secondary job sector to maintain work incentives for workers in a primary sector when adjusting employment.

2 Basic Framework

There are many homogeneous workers and firms in this economy and all they are risk neutral. Suppose that each firm can hire at most one worker. We normalize the measure of total labor force to one. Let u denote the proportion of unemployed workers and e the proportion of employed workers. Thus, we have $e = 1 - u$. A firm is either vacant or filled, and it enters into the search process to find a job seeker. When an employer opens a vacancy, it chooses a wage policy (posting or bargaining). Let η denote the proportion of vacant firms that adopt wage bargaining. η is an endogenous variable. e_p is the number of workers who are employed in jobs that pay a posted wage (call this job "Job p"), and e_b is the number of workers who are employed in jobs that pay a negotiated wage (call this job "Job b"). Therefore, we have $e = e_p + e_b$. We suppose that employed workers in Job p engage in on-the-job search. As is shown later, since workers in Job p receive a wage equivalent to their reservation wage, these workers will have incentives to move to better-compensated jobs (that is, Job b). We further suppose that the firing rate in Job p is higher than that in Job b. This means that workers in Job p experience shorter employment periods compared to Job b. This is one of the features of non-regular jobs. Workers know that there are two types of jobs (Job p and Job b), but they cannot distinguish these jobs before they make a match. Consequently, our model is based on the framework of the random matching model.

2.1 Workers

We represent the job finding rate for job seekers by using the exogenously given matching function: $m(\cdot, \cdot)$. Since only unemployed workers accept job offers made by vacant firms with Job p, the realized number of matches are expressed by $(1 - \eta)\, m(u, v)$. Note that v is the number of vacant firms. $m(u, v)$ exhibits how many matches are created when there are u unemployed workers and v vacant firms. Suppose that this matching function is increasing, concave in each argument, and constant returns to scale. It follows from these settings that the rate of finding Job p (ϕ_p) for unemployed workers is given by $\phi_p = (1 - \eta)\, m(u, v)/u$. On the other hand, not only unemployed workers but also workers employed in Job p are willing to be hired at Job b. The number of matches created in this situation is $\eta\, m(u + e_p, v)$, and the rate of finding Job b (ϕ_b) is described by $\phi_b = \eta\, m(u + e_p, v)/(u + e_p)$. We suppose that ϕ_b is a common job finding rate for both unemployed and employed workers. This formulation of the search process is similar to Pissarides (1994). In Pissarides (1994), employed workers are assumed to accumulate firm-specific skills depending on the duration of their employment, and they earn higher wages as their employment periods become longer. This means that workers who earn sufficiently high wages stay in their current job and stop searching. In contrast, we do not consider the accumulation of human capital and suppose, for simplicity, that all employed workers in Job p look for the opportunity of being hired at Job b. [8)]

8) This seems to be an extreme assumption. However, according to Ronald (2007), in the Netherlands, work experience in a non-standard job (corresponding to a non-regular job) will reduce the possibility of being hired at the standard job (corresponding to the regular job). Workers in non-regular jobs will accumulate some amount of human capital. However, firms are likely to evaluate such work experience

We next characterize the Bellman equations for each group of workers. Unemployed workers receive utility from leisure (or the value of home production), $\bar{w}$, and look for a job. Let $\bar{w}$ be constant. In the random matching framework, job seekers cannot be free to make job applications. The Bellman equation for unemployed workers is given as follows.

$$r\,U \;=\; \bar{w} + \phi_p\,(W_p - U) + \phi_b\,(W_b - U), \tag{1}$$

where W_i is the value of a job i for workers ($i = p,\, b$). Note that job seekers encounter intense competition to find firms with vacancies in Job b because employed workers in Job p join in the search process.

Employed workers belong to either Job p or Job b. Workers who belong to Job b receive a flow wage w_b and do not search on-the-job. The Bellman equation W_b is

$$r\,W_b \;=\; w_b + s\,(U - W_b), \tag{2}$$

where s is an exogenous separation rate. On the other hand, workers employed in Job p receive a wage w_p and are searching on-the-job. The Bellman equation W_p is written as follows:

$$r\,W_p \;=\; w_p + (s + z)\,(U - W_p) + \phi_b\,(W_b - W_p). \tag{3}$$

The last term in (3) represents a change in value when a new job is obtained successfully. The success of an on-the-job search provides employed workers in Job p the new asset value W_b instead of W_p. Since we suppose that job separation in Job p occurs more frequently than in Job b, workers hired at Job p become unemployed at rate $s + z$. We can interpret the parameter z as the possibility of rejection of contract renewal by firms. [9)]

There are several important points to be noted when we construct a model for the market of non-regular workers. First, not all irregular workers want to get regular jobs. According to the 2007 Economic Survey of Japan reported by the Cabinet office, in 2003, around 30% of irregular workers responded that they were unwilling to take a non-regular job. This proportion, while small however, increases year by year. Furthermore, the same economic survey indicates that many young people under age 35 tend to be non-regular workers against their will. From this point of view, our assumption that all employed workers in Job p search on-the-job seems to be immoderate. Even in such an extreme case that firms with Job p face the highest turnover rate and firms with Job b face the highest arrival rate of job applicants, the number of firms with vacancies in Job p is inefficiently greater in the market equilibrium than the socially optimal level. This implies that the number of Job b will become smaller when only a subset of non-regular workers engage in on-the-job search. Thus, we will obtain reasonable results concerning effective policies for

negatively. When we consider skill accumulation in non-regular jobs, we must pay attention to this negative impact on the job finding rate for non-regular workers.

9) Rebick (2005) reviews the recent trends of Japanese labor markets, including the changes in the number of regular and non-regular workers. He indicates that there are two main reasons for such an increase of non-regular workers: the first and most important reason is reducing labor costs; the second reason is employment flexibility. When we incorporate the endogenous job destruction mechanism into our model to capture this flexibility, the additional notions, such as the existence of productivity shock, are necessary. See Mortensen and Pissarides (1994) and Pissarides (2000) in this regard.

reducing non-regular workers. Second, a competitive labor market may be appropriate for representing the instability of non-regular jobs. This is another effective way of dealing with the market of non-regular jobs. [10] However, the result that workers receive wages equal to their marginal productivity does not adequately describe working conditions for non-regular workers. This is because, as stated in the introduction, nonnegligible proportions of part-time workers engage in a mission-critical task, and they do not necessarily receive a fully compensated wage.

2.2 Firms

Firms are classified as either vacant or filled. When opening a vacancy, a firm must choose its wage policy, posting a wage or wage negotiation. Let the value of holding a vacancy of Job p be denoted as V_p, and similarly, denote the value of Job b as V_b. Then these are expressed by

$$r\,V_p \;=\; -\,c_p + q_p\,(J_p - V_p), \tag{4}$$
$$r\,V_b \;=\; -\,c_b + q_b\,(J_b - V_b), \tag{5}$$

where c_i is the job creation cost when an employer opens Job i, and suppose that $c_b > c_p$. Furthermore, q_i $(i = p,\, b)$ is the arrival rate of job seekers to a vacant firm with Job i. Each q_i is defined as follows [11]:

$$q_p \;=\; \frac{m(u,\, v)}{v} \quad \text{and} \quad q_b \;=\; \frac{m(u + e_p,\, v)}{v}.$$

J_i $(i = p,\, b)$ is the value of Job i that is filled with a worker.

The Bellman equation for Job b is written as

$$r\,J_b \;=\; y - w_b + s\,(V_b - J_b), \tag{6}$$

where y is constant productivity in Job b. On the other hand, J_p is expressed by

$$r\,J_p \;=\; \lambda\,y - w_p + (s + z + \phi_b)(V_p - J_p), \tag{7}$$

where we assume $\lambda \in (0,\, 1\,]$. This means that the productivity in Job p is not greater than that in Job b. If workers in Job p engage in almost the same tasks as workers in Job b, λ approaches one. In addition, in (7), the separation rate in Job p includes a job-specific separation rate z and a turnover rate ϕ_b.

10) Davidson et al.(1988) construct a two-sector general equilibrium model with labour market friction. In their model, only one of the labor markets is assumed to be frictional, and the other market is frictionless. We can regard this frictionless market as a market for non-regular jobs.

11) Let v_i be the number of vacancies of Job i $(i = p,\, b)$. Then we have $v = v_p + v_b$, and note that the arrival rate for each type of job is expressed by

$$q_p \;=\; \frac{(1-\eta)\, m(u,\, v)}{v_p} \;=\; \frac{v_p}{v}\,\frac{m(u,\, v)}{v_p} \quad \text{and} \quad q_b \;=\; \frac{\eta\, m(u + e_p,\, v)}{v_b} \;=\; \frac{v_b}{v}\,\frac{m(u + e_p,\, v)}{v_b}.$$

For more precise descriptions, see Pissarides (1994).

We now introduce additional notations. Define $\theta_1 \equiv v/u$ and $\theta_2 \equiv v/(u + e_p)$. Both θ_1 and θ_2 are called labor market tightness. Note that θ_2 is the ratio of vacant firms to job seekers, both unemployed workers and employees in Job p. We then rewrite q_p, q_b, ϕ_p, ϕ_b as $q_p(\theta_1)$, $q_b(\theta_2)$, $\phi_p(\theta_1, \eta)$, $\phi_b(\theta_2, \eta)$. [12)] The assumptions imposed on our matching function result in

$$\frac{d\, q_p(\theta_1)}{d\, \theta_1} < 0, \ \frac{d\, q_b(\theta_2)}{d\, \theta_2} < 0, \ \frac{\partial \phi_p(\theta_1, \eta)}{\partial \theta_1} > 0, \ \frac{\partial \phi_b(\theta_2, \eta)}{\partial \theta_2} > 0.$$

In addition, we suppose that for any $\eta \in (0, 1)$,

$$\lim_{\theta_1 \to 0} q_p(\theta_1) \ = \ \lim_{\theta_2 \to 0} q_b(\theta_2) \ = \ \infty \ \text{ and } \ \lim_{\theta_1 \to 0} \phi_p(\theta_1, \eta) \ = \ \lim_{\theta_2 \to 0} \phi_b(\theta_2, \eta) \ = \ 0,$$
$$\lim_{\theta_1 \to \infty} q_p(\theta_1) \ = \ \lim_{\theta_2 \to \infty} q_b(\theta_2) \ = \ 0 \ \text{ and } \ \lim_{\theta_1 \to \infty} \phi_p(\theta_1, \eta) \ = \ \lim_{\theta_2 \to \infty} \phi_b(\theta_2, \eta) \ = \ \infty,$$

The results in the first line mean that an infinite increase in job seekers makes the arrival rate for vacant firms infinitely large and makes the arrival rate for job seekers approach zero. This relative increase is preferable for vacant firms because the market becomes less competitive for them. On the other hand, job seekers face severe competition due to this increase. A relative increase in vacant jobs has converse impacts on both arrival rates.

3 Wage Determination

When an employer chooses to adopt wage negotiation, a wage is determined by bilateral bargaining after a match is formed. This determination process is described by the asymmetric Nash bargaining problem used in many related studies. The bargained wage is a solution to the following maximization problem:

$$\max_{w_b} \ (W_b - U)^{\beta} (J_b - V_b)^{1-\beta},$$

where $\beta \in (0, 1)$ is the bargaining power of workers. It follows from (1)(2)(5) and (6) that we have

$$w_b \ = \ r\, U + \beta\, (y - r\, V_b - r\, U). \tag{8}$$

In this case, a worker and an employer divide a surplus created by forming a match, and the realized wage level is dependent on the worker's bargaining power.

We next derive a wage that is determined by firms that choose wage posting. We first show that the expected profits of a firm with Job p are monotonically decreasing with the posted wage level. This means that the firm will post a low wage level as long as it provides

12) Notice that each arrival rate can be described as follows:

$$\phi_p(\theta_1, \eta) \ = \ (1 - \eta)\, M(\theta_1), \ \ \phi_b(\theta_2, \eta) \ = \ \eta\, M(\theta_2), \ \ q_p(\theta_1) \ = \ \frac{M(\theta_1)}{\theta_1}, \ \ q_b(\theta_2) \ = \ \frac{M(\theta_2)}{\theta_2},$$

where we define $M(\theta_i)$ as $M(\theta_i) \equiv m(1, \theta_i)$.

job seekers enough utility to accept the offer. It follows from (4) and (7) that expressions for V_p and J_p are rewritten by

$$V_p \;=\; -\frac{c_p}{r + q_p(\theta_1)} + \frac{q_p(\theta_1)}{r + q_p(\theta_1)}\, J_p, \tag{9}$$

$$J_p \;=\; \frac{\lambda\, y - w_p}{r + s + z + \phi_b(\theta_2,\, \eta)} + \frac{s + z + \phi_b}{r + s + z + \phi_b(\theta_2,\, \eta)}\, V_p, \tag{10}$$

and substituting (10) into (9) yields

$$r\, V_p \;=\; \frac{-\,[\, r + s + z + \phi_b(\theta_2,\, \eta)\,]\, c_p + q_p(\theta_1)\,(\lambda\, y - w_p)}{r + s + z + q_p(\theta_1) + \phi_b(\theta_2,\, \eta)}. \tag{11}$$

Equation (11) is obviously a decreasing function of w_p.

Workers accept a wage offer that is greater than or equal to their reservation wage. Since the reservation wage makes workers indifferent to being unemployed or being employed, $W_p = U$ at this wage. This results in the following result:

$$w_p \;=\; \bar{w}. \tag{12}$$

That is, workers' reservation wage is equivalent to their value of leisure. This wage does not reflect the productivity of workers and other marker variables except for $\bar{w}$. An increase in λ, which reduces the productivity gap between Job p and Job b, has no impact on w_p, and the wage difference between these jobs does not shrink. [13)]

By solving for $W_b - U$, we obtain

$$W_b - U \;=\; \frac{w_b - \bar{w}}{r + s + \phi_b(\theta_2,\, \eta)}. \tag{13}$$

(13) and the concrete shape of w_b from (8) provide the expression for $r\,U$,

$$r\,U \;=\; \frac{(r+s)\,\bar{w} + \beta\,\phi_b(\theta_2,\, \eta)\, y}{r + s + \beta\,\phi_b(\theta_2,\, \eta)}, \tag{14}$$

where this equation is evaluated at $V_b = 0$. Since workers are indifferent to being employed at Job p or unemployed, employed workers become better off by being hired at Job b.

It follows from (14) that (8) gives a more concrete expression for w_b as follows:

$$w_b \;=\; \frac{(1-\beta)(r+s)\,\bar{w} + \beta\,[\, r + s + \phi_b(\theta_2,\, \eta)\,]\, y}{r + s + \beta\,\phi_b(\theta_2,\, \eta)}. \tag{15}$$

(12) and (15) result in $w_b > w_p$. Therefore the Job p provides workers low wages and poor job security compared to Job b.

13) It follows from an OECD report (2008) that (i) wage payments in non-regular jobs are relatively low compared to those in regular jobs, irrespective of gender (OECD (2005) suggests that the hourly wage rate and weekly earnings for part-time employees in Japan are significantly lower than in other OECD countries); (ii) this wage gap increases as employment periods become longer. In our model, a posted wage does not reflect a change in market conditions because firms with job vacancies that adopt a posting mechanism commit to paying this predetermined wage once the vacancies are filled. Therefore, our description of non-regular jobs is not unreasonable.

4 Job Creation Conditions

We suppose that the free entry/exit condition holds. The labor market tightnesses $(\theta_1,\, \theta_2)$ and the proportion of firms with Job b (η) are determined by conditions $J_p = 0$ and $J_b = 0$. We pay attention to these two conditions because this captures a situation in which both types of jobs coexist in equilibrium. The expressions of these conditions are derived in this section.

It follows from (11) and (12) that the value of a vacant firm with Job p can be written as

$$r\,V_p \;=\; \frac{-\,[\,r+s+z+\phi_b(\theta_2,\,\eta)\,]\,c_p + q_p(\theta_1)\,(\lambda\,y-\bar{w})}{r+s+z+q_p(\theta_1)}.$$

Under $V_p = 0$, the following expression is obtained:

$$\frac{c_p}{q_p(\theta_1)} \;=\; \frac{\lambda\,y-\bar{w}}{r+s+z+\phi_b(\theta_2,\,\eta)+\phi_b(\theta_2,\,\eta)}. \tag{16}$$

Similarly, we derive the same expression for Job b as (16). Computing $J_b - V_b$ by using (5) and (6) and substituting this into (5) yields

$$r\,V_b \;=\; \frac{-\,(r+s)\,c_b + q_b(\theta_2)\,(y-w_b)}{r+s+q_b(\theta_2)}.$$

Then the condition $V_b = 0$ is equivalent to

$$\frac{c_b}{q_b(\theta_2)} \;=\; \frac{(1-\beta)(y-\bar{w})}{r+s+\beta\,\phi_b(\theta_2,\,\eta)}. \tag{17}$$

Expressions (16) and (17) are the job creation conditions in this model. These conditions indicate that the discounted expected costs of having a vacancy (LHS) are equal to the maximized future profits (RHS).

Equation (16) determines θ_1, and equation (17) determines θ_2. These market tightness values are represented as functions of η. (16) and (17) provide the relationship between θ_i and η for each i. To describe these relationships, we rewrite job creation conditions as follows:

$$\frac{[\,r+s+z+\phi_b(\theta_2,\,\eta)\,]\,c_p}{q_p(\theta_1)} \;=\; \lambda\,y-\bar{w}, \tag{16'}$$

$$\frac{[\,r+s+\beta\,\phi_b(\theta_2,\,\eta)\,]\,c_b}{q_b(\theta_2)} \;=\; (1-\beta)(y-\bar{w}). \tag{17'}$$

Partial derivatives of $(17')$ with respect to θ_2 and η are given by

$$\frac{\partial \text{ LHS of } (17')}{\partial\,\theta_2} \;>\; 0, \quad \frac{\partial \text{ LHS of } (17')}{\partial\,\eta} \;>\; 0.$$

The results and assumptions for q_b and ϕ_b yield a unique relationship between θ_2 and η such that $d\,\theta_2\,/\,d\,\eta < 0$.

We next derive from (16′) the relationship between θ_1 and η. By taking the indirect effect of η through θ_2 into account, partial derivatives of the left hand side of (16′) with respect to θ_1 and η are

$$\frac{\partial \text{ LHS of (16}')}{\partial \theta_1} > 0, \quad \frac{\partial \text{ LHS of (16}')}{\partial \eta} = \frac{c_p \, d\,\phi_b(\theta_2, \eta)/d\,\eta}{q_p(\theta_1)}.$$

Concerning $d\,\phi_b/d\,\eta$, we have the following lemma.

Lemma 1
If the bargaining power of workers is sufficiently low, ϕ_b is increasing in η.

Lemma 1 implies that an increase in the proportion of vacancies of Job b does not necessarily increase the arrival rate for this job. This rise increases the possibility of meeting Job b in the search process, but on the other hand, it reduces the tightness θ_2 because the above increase in the possibility of meeting gives workers a stronger position in wage bargaining, discouraging firms from making a match. Thus, the total impact on the meeting possibility ϕ_b is ambiguous without any conditions. The condition for the worker's bargaining power makes the latter effects sufficiently small.

Proposition 1
If Lemma 1 holds, θ_1 and θ_2 that satisfy the job creation conditions are uniquely represented by functions of η. Furthermore, for each θ_i $(i = 1, 2)$, we have $d\,\theta_i/d\,\eta < 0$.

In later sections, let θ_i be denoted as $\theta_i(\eta)$ for each i. In addition, we rewrite q_j and ϕ_j $(j = p, b)$ as $q_j(\eta)$ and $\phi_j(\eta)$.

5 Steady State Equilibrium

5.1 Steady State Unemployment Rate and Employment Composition

Flow conditions for workers are needed to describe the steady state equilibrium completely. Transitions of unemployed worker flows are given by

$$\dot{u} = [\,(s+z)\,e_p + s\,e_b\,] - [\,\phi_p(\eta) + \phi_b(\eta)\,]\,u. \tag{18}$$

The first term in (18) is the inflow to the unemployment pool, and the second term represents the outflow from the unemployment pool. At steady state, $\dot{u} = 0$, and we have the steady state unemployment rate as follows:

$$u = \frac{s + z\,e_p}{s + \phi_p(\eta) + \phi_b(\eta)}, \tag{19}$$

where we use the fact that $e_p + e_b = e = 1 - u$. On the other hand, the transitions of workers who are hired at Job p are expressed as

$$\dot{e}_p = \phi_p(\eta)\,u - [\,(s+z)\,e_p + \phi_b(\eta)\,e_p\,]. \tag{20}$$

The second term represents the outflow from the pool of this type of workers. This outflow is composed of two factors of separation: (i) a flow into unemployment and (ii) a transition to Job b. Under the steady state condition $\dot{e}_p = 0$, solving for e_p yields

$$e_p = \frac{\phi_p(\eta)\, u}{s + z + \phi_b(\eta)}. \tag{21}$$

Substituting (21) into (19) results in

$$u = \frac{s\,[\, s + z + \phi_b(\eta)\,]}{[\, s + \phi_b(\eta)\,]\,[\, s + z + \phi_p(\eta) + \phi_b(\eta)\,]}. \tag{22}$$

Furthermore, substituting (22) into (21) yields

$$e_p = \frac{s\,\phi_p(\eta)}{[\, s + \phi_b(\eta)\,]\,[\, s + z + \phi_p(\eta) + \phi_b(\eta)\,]}. \tag{23}$$

When the market tightnesses θ_1 and θ_2 are determined, u and e_p are characterized as unique functions of η. In the next subsection, we derive an equation and conditions for characterizing a unique value of η.

5.2 Existence and Uniqueness of the Steady State equilibrium

We first note that definitions of θ_1 and θ_2 give us

$$\theta_2(\eta) = \frac{v}{u + e_p} = \frac{\theta_1(\eta)\, u}{u + e_p} = \frac{\theta_1(\eta)}{1 + e_p/u},$$

where $\theta_1(\eta)$ is the unique solution to (16′). It follows from (21) that we have

$$\theta_2(\eta) = \theta_1(\eta) \Bigg/ \left[1 + \frac{\phi_p(\eta)}{s + z + \phi_b(\eta)} \right] \equiv \frac{\theta_1(\eta)}{\Phi(\eta)}. \tag{24}$$

$\theta_2(\eta)$ is determined by (17′), and η has an effect on the RHS of (24) through θ_1, u and e_p. Equation (24) is a final condition for characterizing a steady state equilibrium. The main purpose of this subsection is to find conditions under which there exists a unique $\eta \in (0,\, 1)$ satisfying (24).

It can be noticed that the following facts hold: $d\,\phi_p/d\,\eta < 0$ and $d\,\phi_b/d\,\eta > 0$. The former fact results from the definition of ϕ_p, and the latter is a result of Lemma 1. These facts and the definition of $\Phi(\eta)$ yield $\Phi'(\eta) < 0$. The function $\Phi(\eta)$ denotes the ratio $1 + e_p/u$ and is decreasing in η. Since a rise in η increases ϕ_b, job seekers can make a match more easily than before. On the other hand, the arrival rate for unemployed workers ϕ_p declines with the increase in η. This effect makes it difficult for the unemployed to find a job. Since making a match with Job b is more competitive than Job p, these effects result in a decrease in e_p and an increase in u. Thus, the ratio e_p/u falls with an increase in η.

Based on the results concerning ϕ_p, ϕ_b and $\Phi(\eta)$, we proceed to examine the existence of a unique η satisfying (24). Let each side of (24) be regarded as a function of η (denote the RHS of (24) as $T(\eta)$). Then its LHS is a decreasing function of η. On the other hand, differentiating the RHS of (24) results in

$$\frac{d\,\text{RHS of (24)}}{d\,\eta} = \frac{\Phi(\eta)\, d\,\theta_1/d\,\eta - \theta_1\, \Phi'(\eta)}{\Phi^2(\eta)}.$$

A sign of this derivative is ambiguous because $d\,\theta_1/d\,\eta < 0$ and $\Phi'(\eta) < 0$. However, notice that we have the following lemma.

Lemma 2
Under condition (25), *the RHS of* (24) *is an increasing function of* η.

$$1 - \frac{(s + z + \phi_b(\eta))M(\theta_2)}{[\, r + s + z + \phi_b(\eta)\,]\, M(\theta_1)} \geq \xi(\theta_1). \tag{25}$$

The proof of this lemma is given in Appendix B. [14)]

Consequently, if (25) holds, the RHS of (24) is increasing in η and the LHS of (24) is decreasing in η. This means that there exists a unique η satisfying (24), provided that boundary conditions at $\eta = 0$ and $\eta = 1$ are met. We provide a precise proof in Appendix C.

Proposition 2
When the productivity of Job p is sufficiently large, and the job creation cost and the separation rate, which are specific to this job, satisfy

$$\frac{[\, r + s + \beta\, M(\theta_2)\,]\, c_b}{r + s + z + M(\theta_2)} > c_p, \tag{26}$$

for any fixed η, there exists a unique η which characterizes the equilibrium that makes firms indifferent between posting a wage and wage negotiation.

These conditions for parameters exhibit the relationship between θ_2 and $T(\eta)$ at $\eta = 0$ and $\eta = 1$. The determination of this unique η is described by Figure 1. Two boundary conditions ensure that there is one intersection point of two curves in the interval (0, 1). These conditions require that the productivity gap between Job p and Job b be small and the job separation rate specific to Job p be neither too high nor too low such that the condition (26) is satisfied. It should also be noted that if the job creation costs of Job b is less than

14) The condition of ξ required for Lemma 2 is satisfied under some popular form of a matching function. Suppose that a matching function is represented by $m(u,\, v) = A\, u^{1/2}\, v^{1/2}$ $(A > 0)$. Then we have $M(\theta_1) = A\, \theta_1^{1/2}$. A similar result is obtained in the analysis of $M(\theta_2)$. This specification implies that the elasticity $\xi(\theta_1)$ is always independent of θ and is equal to 1/2. Therefore, if the LHS of (25) is greater than 1/2, the inequality (25) is satisfied. Since the LHS $\geq 1/2$ is equivalent to $M(\theta_1)/M(\theta_2) \geq 1/2$ for a sufficiently small r, (25) is written as

$$(\theta_1/\theta_2)^{1/2} = [\,(u + e_p)/u\,]^{1/2} \geq 1/2.$$

When we set $u = 0.04$ and $e_p = 0.3$, the above equality is automatically satisfied. These parameter values are consistent with the recent labor-market conditions in Japan. Thus, the assumption in Lemma 2 is likely to be satisfied in reality.

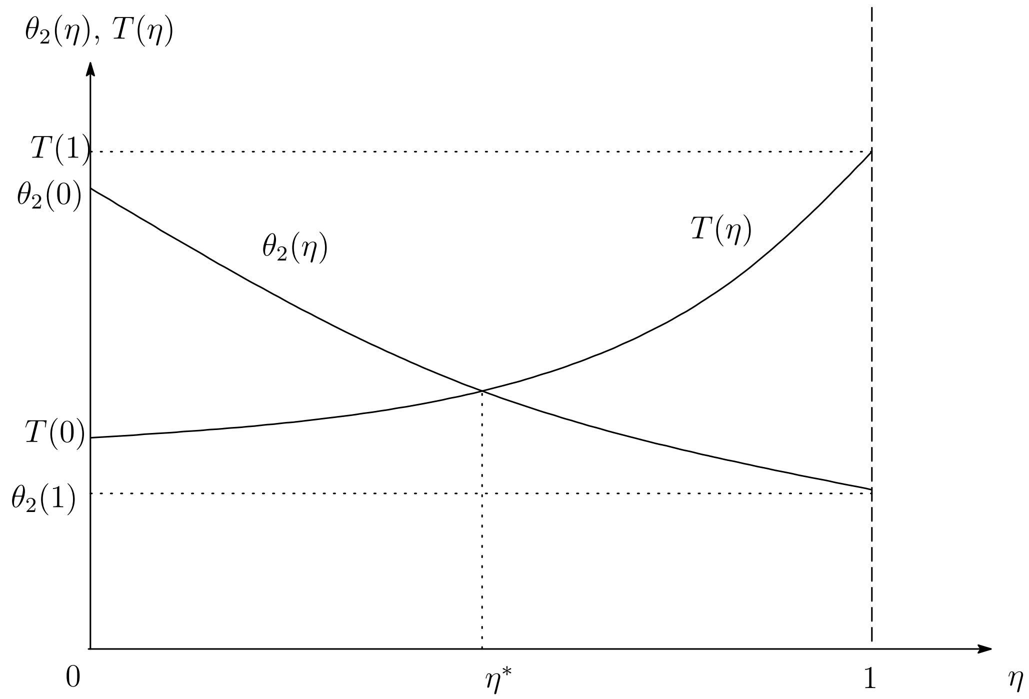

Figure 1: The equilibrium value of η

those of Job p, the condition (26) is never satisfied. It is then difficult to characterize the equilibrium with the coexistence of these types of workers. In Acemogle (2001), there are good jobs and bad jobs in the economy. Good jobs have greater productivity than bad jobs. Both jobs produce intermediate goods and the technology of producing a final good is given by a CES function. If the job creation costs of good jobs are greater than those of bad jobs, the existence and the uniqueness of equilibrium depend on a sign of the elasticity of substitution, ρ (for example, there is a unique equilibrium when good jobs and bad jobs are gross complements: $\rho \leq 0$). On the other hand, we do not consider such a substitutability between Job b and Job p in our model. Thus we must specify the values of parameters which characterize a steady state equilibrium. In the context of non-regular jobs, we can say that if (i) non-regular workers take charge of similar tasks as regular workers and (ii) the employment period in Job p is moderately set, these types of workers coexist in the labor market. [15)]

It follows from the analyses in this section that the unique equilibrium in this model is fully described by variables $\{w_p, w_b, \theta_1, \theta_2, u, e_p, \eta\}$ and equations (12), (15), (16′), (17′), (22),(23) and (24). We first obtain the value of η from (24). After that, θ_1 and θ_2 result from (16′) and (17′). (22) and (23) determine u and e_p, and finally, w_p and w_b are obtained from (12) and (15) (w_p is constant).

15) In a real labor market, there are various types of non-regular jobs that differ with respect to their employment duration. According to the 2007 Employment Status Survey published by the Ministry of Health, Labour and Welfare in Japan, for example, the number of daily workers whose employment duration is between one day and one month is fairly small (about 2.3% of the total labor force). We must keep in mind that many of these workers endure a minimum standard of living and their proportion rises gradually.

6 Properties of the Equilibrium

In this section, we first show that the ratio η determined through market mechanisms is lower than the socially efficient level that maximizes social surplus. Based on this analysis, policies of increasing η and the ratio e_b/e_p and improving social surplus are examined. We focus on three cases: (i) the productivity gap between jobs shrinks; (ii) employment duration in Job p becomes shorter; and (iii) the minimum wage is set above the wage paid by firms with Job p.

6.1 Social Efficiency

According to Hosios (1990) and Pissarides (2000), the social surplus (SS) in our model can be calculated as follows:

$$\text{SS} \ = u\,\bar{w} + e_p\,\lambda\,y + e_b\,y - (1-\eta)\,\theta_1\,u\,c_p - \eta\,\theta_2(u+e_p)\,c_b. \tag{27}$$

Many related studies focus on the situation in which the social surplus is maximized, subject to constraints concerning dynamic equations of unemployed or employed workers. Here, we do not find a socially efficient value of η. Instead, we show that η, as determined in the decentralized equalities, is not efficient, and in the later subsections, we intend to examine what policies can improve the composition of jobs in the labor market and increase the social surplus.

We follow the same procedures as Acemogle (2001): if the decentralized equilibrium were socially efficient, (27) with the equilibrium values of u, e_p and e_b would have a zero derivative. However, studies in this subsection show that the number of vacancies in Job b is less than the socially efficient level (that is, there are too many non-regular jobs in the decentralized equilibrium). By substituting (22) and (23) into u and e_p respectively, (27) can be rewritten as follows (let $r \to 0$):

$$\text{SS} \,=\, \frac{s\,\bar{w}}{s+\beta\,\phi_b(\eta)} + \frac{\beta\,\phi_b(\eta)\,y}{s+\beta\,\phi_b(\eta)}, \tag{28}$$

where we have used the fact that $e_b = \phi_b(\eta)/[\,s+\phi_b(\eta)\,]$. Furthermore, expressions for c_p and c_b given by (16′) and (17′) are incorporated into (27). The differentiation of (28) with respect to η yields

$$\frac{d}{d\,\eta}\left(\frac{1}{s+\beta\,\phi_b(\eta)}\right) s\,\bar{w} + \frac{d}{d\eta}\left(\frac{\phi_b(\eta)}{s+\beta\,\phi_b(\eta)}\right)\beta\,y \,=\, \frac{s\,\beta\,(y-\bar{w})}{[\,s+\beta\,\phi_b(\eta)\,]^2}\,\frac{d\,\phi_b(\eta)}{d\,\eta} \,>\, 0,$$

where we treat the nontrivial case $y > \bar{w}$.

Proposition 3
In the decentralized equilibrium formed by a market mechanism, a smaller number of vacancies of Job b are created in comparison with the socially efficient level. In other words, a policy of raising the proportion of Job b increases the social surplus.

It is well-known that ex post wage bargaining can not internalize the impact of the search externality because bargainers do not take into account this externality when they determine a wage level. Acemogle (2001) shows that the proportion of low productivity jobs is inefficiently high in equilibrium. In our model, vacancies of Job p (corresponding to bad jobs) are created at a rate greater than the socially efficient level. This result is due to the similar reason as indicated in the related literatures including Acemogle (2001).

Proposition 3 indicates that a decentralized market equilibrium is accompanied by inefficiently large numbers of non-regular jobs. Therefore, in order to improve social efficiency, the government should implement policies that raise the proportion of regular jobs (η in our model). We notice that such an inefficient creation of non-regular jobs is realized even when we do not take into account payments for job training or social securities, which increase labor costs for employers who hire regular workers. This indicates that an excessive number of non-regular jobs will be created if we incorporate these factors into the model.

6.2 The Impact of Reducing the Productivity Gap between Jobs

In this subsection, we examine the effect that a rise in λ has on η^*, the ratio e_b/e_p and the social surplus. First, the rise in λ increases the RHS of (16′). To continue to satisfy the zero profit condition, θ_1 must increase for any fixed η. A high level of λ means that firms with Job p receive higher revenues than before. This gives potential employers incentive to participate in the labor market and increases θ_1.

Concerning the impact on η, since λ has only an indirect effect on θ_2, η is affected by a change in λ only through θ_1. For any fixed η, partial differentiation of $T(\eta)$ with respect to θ_1 yields $\partial T(\eta)/\partial\theta_1 > 0$. This means that $T(\eta)$ moves upwards by an increase in θ_1. Since θ_2 is unchanged, a rise in λ decreases η ($\eta^* \to \eta'$ at Figure 2). Since an increase in the productivity of Job p does not increase wage payments, this productivity increase is equivalent to profits of a firm. This is appealing for other firms and raises the proportion of Job p.

We next examine how a rise in λ affects the number of workers hired at Job b, e_b. Since we have $e_b = e - e_p = 1 - u - e_p$, the expression for e_b results from (22) and (23) as follows:

$$e_b \;=\; 1 - \frac{s}{s + \phi_b(\eta)} \;=\; \frac{\phi_b(\eta)}{s + \phi_b(\eta)}. \tag{29}$$

Notice that $\phi_b(\theta_2)$ is decreasing in λ because of $d\,\phi_b/d\,\eta > 0$ derived in the previous section. We then obtain from (29) that $\partial\, e_b\,/\,\partial\,\lambda < 0$. Therefore, an increase in the productivity of Job p decreases the number of workers in Job b. Since non-regular jobs become more productive than before, many employers have incentive to create this type of jobs, and the number of non-regular workers increases (as indicated by Labour force survey 2008 reported by MIC).

On the other hand, what impact does a rise in λ have on the ratio e_b/e_p ? This ratio is derived from (23) and (29) and is expressed as

$$\frac{e_b}{e_p} \;=\; \frac{\phi_b(\eta)\,[\,s + z + \phi_p(\eta) + \phi_b(\eta)\,]}{s\,\phi_p(\eta)}. \tag{30}$$

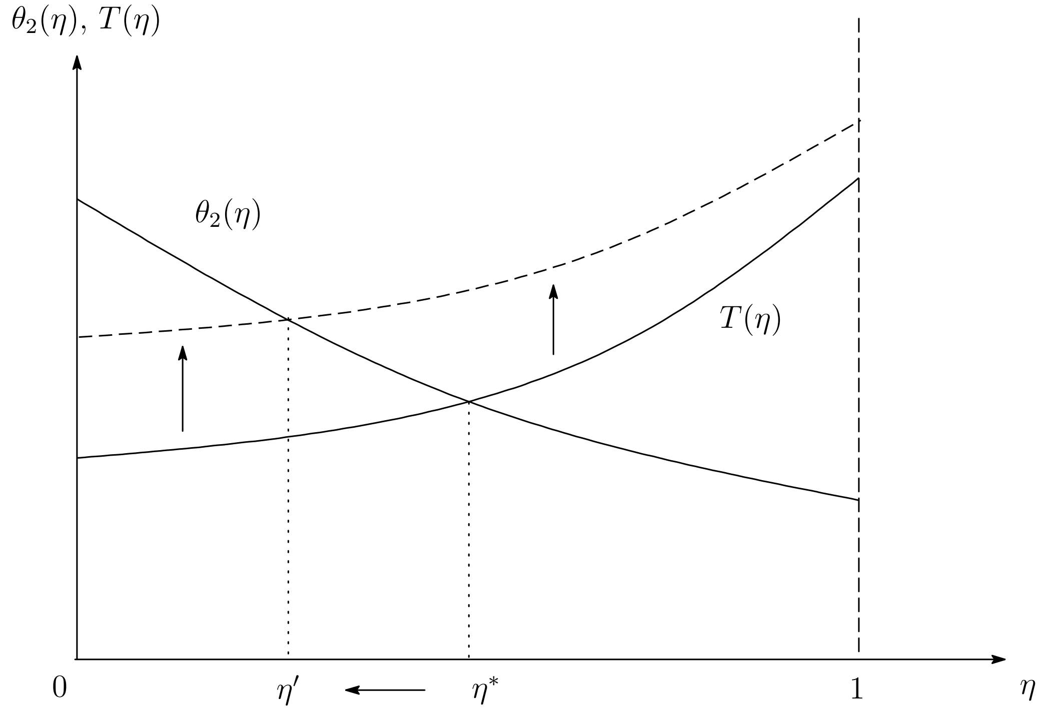

Figure 2: An impact of a rise inλ on η

Partially differentiating (30) with respect to λ yields

$$\frac{\partial}{\partial \lambda}\left(\frac{e_b}{e_p}\right) < 0.$$

This results from the facts that $\partial \theta_1/\partial\lambda > 0$, $\partial\eta/\partial\lambda < 0$, $\partial \phi_p/\partial \lambda > 0$ and $\partial \phi_b/\partial \lambda < 0$. Thus, a higher value of λ not only decreases the number of workers hired at Job b, but also reduces the ratio of employed workers e_b/e_p. The rise in productivity increases profits of firms with Job p. However, workers hired at this job receive no benefit from this productivity increase. Many potential firms have incentive to open a vacancy in the form of Job p, and more workers become low-wage earners. Since the social surplus given by (28) is decreasing in η, the rise in λ reduces the social surplus. As a result, an increase in the productivity of non-regular jobs, or expanding the number of jobs that are targets of deregulations in employing non-regular workers, will have an unfavorable impact on the economy.

Proposition 4

An increase in the productivity (λ) *of Job p reduces the proportion* (η) *of firms that choose to open a vacancy with Job b and the number of workers* (e_b) *hired at Job b. Furthermore, this increase also reduces the ratio* e_b/e_p *and the social surplus.*

The results obtained in Proposition 4 are highly intuitive and not surprising. But studies in the later subsections are important for finding effective ways to raise the proportion of regular workers and to improve social efficiency.

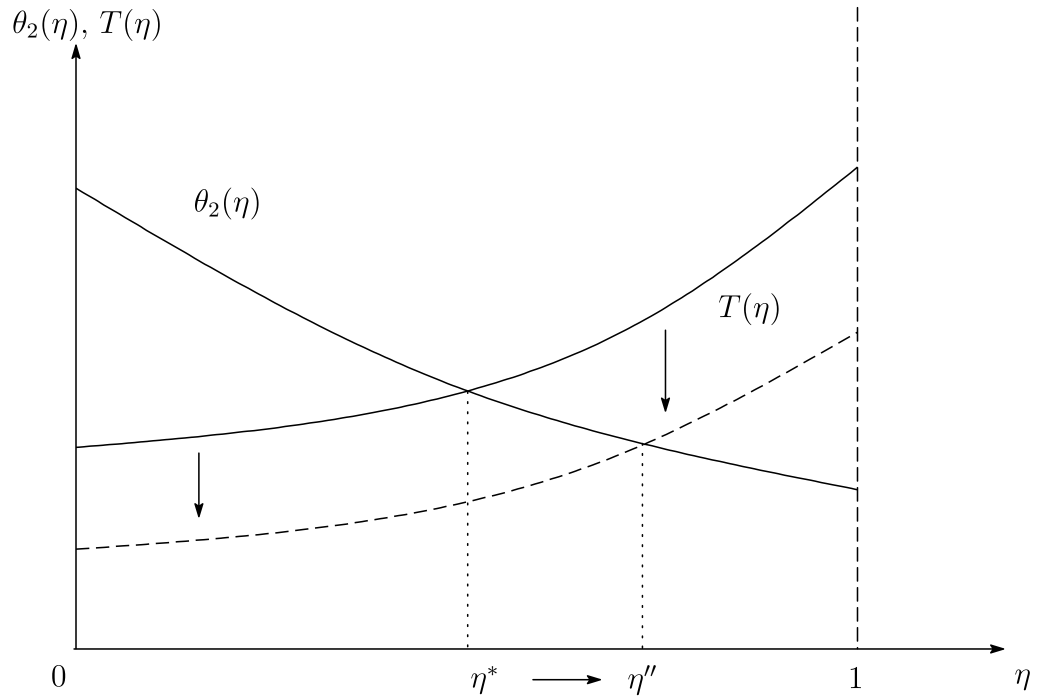

Figure 3: An impact of a rise in z

6.3 The Impact of Shorter Employment Period in Job p

In this subsection, we examine the impact of a change in z on η^*, e_b and e_b/e_p. A rise in z increases the LHS of (16′). Then θ_1 must fall in order to maintain the job creation condition for Job p.

The RHS of (24) depends on z both directly and indirectly through θ_1. Based on these effects, $\partial T(\eta) / \partial z$ has an ambiguous sign. By letting the interest rate become sufficiently small, however, $\partial T(\eta) / \partial z$ is negative. This means that the rise in z increases η ($\eta^* \to \eta''$ at Figure 3). In addition, the number of workers in Job b increases ($\partial e_b/\partial z > 0$). Since a higher separation rate leads to a higher possibility of vacancy for firms, they are reluctant to have a vacancy in the form of Job p. This decision by firms results in an increase in η.

We next examine how the rise in z affects the ratio e_b/e_p. It follows from (30) that we have

$$\frac{\partial}{\partial z}\left(\frac{e_b}{e_p}\right) > 0,$$

where the facts $\partial \phi_b(\eta)/\partial z > 0$ and $\partial \phi_p(\eta)/\partial z < 0$ are used to obtain this result. Thus, the more unstable Job p becomes, the more the ratio of the number of workers in Job b to the number of workers in Job p goes up. [16)] By the same logic as in the previous

16) It seems that if a higher z increases the number of regular jobs, increasing the number of more unstable daily jobs would be a plausible policy for improving an employment structure. However, this logic is not correct. This is because Proposition 2 requires that the productivity of Job p must be sufficiently high and the separation rate of this job must be neither too high nor too low. Thus, our attention is given to more productive and stable jobs within non-regular jobs.

subsection, we have $\partial \, \text{SS} \, / \, \partial \, z > 0$. That is, a rise in z increases the social surplus.

Finally, we investigate the impact of a change in z on the wage gap $w_b - \bar{w}$ between jobs. This gap is expressed by

$$w_b - \bar{w} \; = \; \frac{\beta \, [\, r + s + \phi_b(\eta) \,] \, (y - \bar{w})}{r + s + \beta \, \phi_b(\eta)}. \tag{31}$$

It follows from (31) that a rise in z affects the wage gap $w_b - \bar{w}$ only through ϕ_b. Noting that $d\,(w_b - \bar{w})/d\,\phi_b > 0$. Connecting this fact with $\partial\,\phi_b/\partial\,z \; = \; (d\,\phi_b/d\,\eta)(\partial\,\eta/\partial\,z) > 0$ yields $\partial\,(w_b - \bar{w})/\partial\,z > 0$. As a result, the rise in z enlarges the wage differential between jobs, while this rise increases both the proportion of vacancies with Job b and the ratio of employed workers e_b/e_p. A higher ϕ_b means that even if workers in Job b lose their current jobs, they will return to the same type of job sooner than before. This makes workers' bargaining position stronger and raises the wage paid in Job b.

Proposition 5

When Job p becomes more unstable, the proportion of Job b (η), the number of workers hired at Job b (e_b), the ratio of employed workers (e_b/e_p) and the social surplus increase. On the other hand, this change enlarges the wage gap between jobs.

6.4 An Impact of Raising Minimum Wages

Suppose that the mandated minimum wage, w_m, is set above the wages paid in Job p. This is described as $w_m > \bar{w}$ ($w_p = \bar{w}$ by (12)).

The job creation condition for Job p, evaluated at $w_p = w_m$, is written as

$$\frac{[\, r + s + z + \phi_b(\eta) \,] \, c_p}{q_p(\eta)} \; = \; \lambda \, y - w_m.$$

Since $w_m > \bar{w}$, θ_1 satisfying this equation must be smaller than the value determined by (16′). This reflects less participation by potential firms. It follows from (24) that this decline in θ_1 raises η and, therefore, increases ϕ_b. We then obtain $\partial e_b/\partial w_m > 0$. Furthermore, by using similar arguments to Subsection 6.2, the introduction of the minimum wage also increases the ratio e_b/e_p. Thus, the presence of the legally binding minimum wage will increase the number of regular workers and its ratio to non-regular workers (this also increases social surplus).

On the other hand, the introduction of the minimum wage has an ambiguous effect on the wage gap between jobs. This is because an increase in w_m has positive impacts on both w_b and w_m:

$$\frac{\partial\,(w_b - w_m)}{\partial\,w_m} \; = \; \frac{d\;w_b}{d\;\phi_b(\eta)} \, \frac{d\,\phi_b}{d\,\eta} \, \frac{\partial\,\eta}{\partial\,w_m} - 1.$$

Summarizing the results obtained in this subsection yields the following proposition.

Proposition 6

The presence of the legally binding minimum wage increases the proportion of Job b (η), the number of workers in Job b (e_b), the ratio of employed workers (e_b/e_p), and the social surplus. On the other hand, we cannot know how this minimum wage affects the wage gap between jobs.

6.5 Discussion

This subsection first gives additional explanations for the above three propositions. In Subsection 6.2, we confirmed that an increase in the productivity of Job p, which reduces the productivity gap between jobs, benefited firms having this job. Recently, in Japan, reforms of the Worker Dispatch Law executed in 1999 and 2004 enable employers to employ persons as non-regular workers in various kinds of occupations and for longer periods. Such revisions of the law increase the number of dispatched people (who have some sort of skills), and, these days, these workers account for $7 \sim 8\%$ of the total number of non-regular workers in Japan. The average number of these people in 2007 was 5 times as much as the number in 1999 (these data result from the Labour Force Survey reported by MIC). The results of Proposition 4 are compatible with the impact of the deregulation policies.

In Subsection 6.3, the impacts of a change in the stability of Job p were investigated. A fixed-term employment contract is one of the important features of non-regular jobs. Workers in non-regular jobs do not always renew their contracts, and sometimes they may lose their current jobs. This means that non-regular jobs are useful for employers because of their flexibility, while non-regular workers suffer a high risk of losing their jobs compared to regular workers. The results in Proposition 5 that a rise in z increases the number and the proportion of Job b (regular jobs) indicate that non-regular jobs with a long employment period are not socially preferable because the existence of such jobs keeps workers in low-paying jobs for a long time. According to Proposition 3, the decentralized market mechanism creates more low-paying jobs than the socially efficient level. Therefore, intervention by the government is justified, and shortening employment periods in non-regular jobs will be an effective policy for increasing regular jobs. We notice, however, that this policy may further expand the wage gap between regular workers and non-regular workers.

In Subsection 6.4, Proposition 6 states that raising the minimum wage will also be an effective way to increase regular jobs and social surplus. For a firm with Job p, this increase only boosts the labor costs for having this job determinately. This makes employers reluctant to open a vacancy in the form of Job p. On the other hand, the proportion of firms with Job b rises along with an increase in the minimum wage. However, this increase in the proportion affects the success rate of finding Job b for job seekers and increases the wage w_b. Thus, higher minimum wages result in higher wages in Job b. According to the OECD (2006), Japan has the third lowest hourly minimum wage (as a percentage of the net average wage) among 21 OECD countries. The proportion of the minimum wage to the average wage in Japan is only half of the proportion in Ireland. This suggests that there is room for improving the composition of jobs by raising the minimum wage in the Japanese economy. Although the impact of a rise in the minimum wage on the wage gap between jobs is ambiguous, this policy may be more desirable than shortening the employment period of Job p because the latter policy unambiguously expands the wage difference. Finally, we note that polices of increasing the number and the proportion of Job b improve social efficiency. These policies will consequently be supported from a social point of view.

These propositions suggest that policy makers can improve the composition of jobs by increasing social surplus, but this improvement will also require some degree of sacrifice. Such policies increase the number of people who look for a well-paid job and receive greater utility than non-regular and unemployed workers. On the other hand, these policies may enlarge the wage differentials between regular and non-regular workers. Furthermore, the above policies have an ambiguous effect on unemployment rate, and in some cases, they may increase unemployment. [17] Thus, we should keep in mind that an increase in the number of regular jobs and regular workers is founded on sacrifices, such as a relative decline in the wages of non-regular workers or an increase in unemployment.

7 Conclusions

We have characterized an equilibrium in which some firms post a wage and others bargain with a worker, focusing on a change in the composition of these firms. This is a natural extension of the standard job search model, where such an extension provides a framework for studying issues related to the differences in working conditions between regular workers and non-regular workers. In this paper, we suppose that employed workers hired at Job p search for a well-paid job (Job b). The following results are obtained. (i) There exists a unique equilibrium in which some firms post a wage and others bargain with a worker under some parameter values. (ii) The decentralized market mechanism generates too many vacancies of Job p. This does not maximize the social surplus, and an additional creation of Job b is necessary to achieve a socially efficient allocation of jobs. (iii) As the productivity difference between jobs becomes smaller, more firms choose to post a wage. (iv) A high separation rate specific to Job p increases the proportion of Job b and the number and the ratio of employed workers hired at Job b. Furthermore, this enlarges the wage gap between jobs. (v) Setting a minimum wage that is greater than the wage paid in Job p has the same effect as a higher separation rate in (iv). However, the introduction of this minimum wage has an ambiguous effect on the wage gap. It follows from (iv) and (v) that policies on these items improve social efficiency by changing the composition of jobs and workers in the labor market. As a result, this paper suggests that policy makers should shorten the employment period of Job p or raise the minimum wage level to make employers reluctant to hire workers into non-regular jobs. Our model also indicates that these policies have different impacts on the wage gap between jobs.

Appendix

A. Proof of Lemma 1

First, $d\phi_b/d\eta$ is expressed as follows:

$$\frac{d\phi_b}{d\eta} = M(\theta_2) + \eta M'(\theta_2)\frac{d\theta_2}{d\eta} = M(\theta_2)\left[1 + \frac{\theta_2 M'(\theta_2)}{M(\theta_2)}\left(\frac{\eta}{\theta_2}\frac{d\theta_2}{d\eta}\right)\right]. \tag{A.1}$$

17) If $d\phi_p/d\eta + d\phi_b/d\eta > 0$, we have $du/d\eta < 0$. Thus, a rise in the minimum wage reduces the unemployment rate, since this policy increases the proportion η.

Below, we show that a sign of this formula becomes positive under certain conditions. Since $d\theta_2/d\eta$ is calculated from (17′), we have

$$\frac{d\theta_2}{d\eta} = -\frac{\beta M^2(\theta_2)}{\beta\eta M^2(\theta_2) + (r+s)[M(\theta_2) - \theta_2 M'(\theta_2)]}.$$

Then (A.1) becomes

$$\frac{d\phi_b}{d\eta} = \frac{\beta\eta M^2(\theta_2) + (r+s)[M(\theta_2) - \theta_2 M'(\theta_2)] - \beta\eta M(\theta_2) M'(\theta_2)}{\{\beta\eta M^2(\theta_2) + (r+s)[M(\theta_2) - \theta_2 M'(\theta_2)]\}^2}.$$

This is equal in sign to

$$\beta\eta M'(\theta_2)\left[1 - \frac{M'(\theta_2)}{M(\theta_2)}\right] + (r+s)\left[1 - \frac{\theta_2 M'(\theta_2)}{M(\theta_2)}\right].$$

Although we cannot know whether $M(\theta_2)$ is greater than $M'(\theta_2)$ for any θ_2, it is easy to show that $M'(\theta_2)/M(\theta_2)$ is decreasing in θ_2 because of the concavity of $M(\cdot)$. Furthermore, it follows from (17′) that $\partial\theta_2/\partial\beta < 0$, a lower β leads to the smaller value of $M'(\theta_2)/M(\theta_2)$. For a sufficiently small β, (A.1) is obviously positive and close to 1. We complete the proof.

B. Proof of Lemma 2

In subsection 5.2, differentiating the RHS of (24) with respect to η results in

$$\frac{d\,\text{RHS of (24)}}{d\eta} = \frac{\Phi(\eta)\, d\theta_1/d\eta - \theta_1 \Phi'(\eta)}{\Phi^2(\eta)}. \tag{B.1}$$

We show that this equation has a positive sign when (25) is satisfied.

It follows from (16′) that the partial differentiation of θ_1 with respect to η is given by [18)]

$$\frac{d\theta_1}{d\eta} = \frac{q_p(\theta_1)\, d\phi_b(\eta)/d\eta}{[r+s+z+\phi_b(\eta)]\, q_p'(\theta_1)}, \tag{B.2}$$

where

$$\frac{\partial\ \text{LHS of (16′)}}{\partial\theta_1} = -\frac{c_p[r+s+z+\phi_b(\eta)]\, q_p'(\theta_1)}{q_p^2(\theta_1)}, \quad \frac{\partial\ \text{LHS of (16′)}}{\partial\eta} = \frac{c_p\, d\phi_b(\eta)/d\eta}{q_p(\theta_1)}.$$

Since $q_p(\theta_1) = M(\theta_1)/\theta_1$ and $q_p'(\theta_1)$ is expressed by

$$q_p'(\theta_1) = \frac{d}{d\theta_1}\left(\frac{M(\theta_1)}{\theta_1}\right) = \frac{\theta_1 M'(\theta_1) - M(\theta_1)}{(\theta_1)^2},$$

then $q_p(\theta_1)/q_p'(\theta_1)$ becomes

$$\frac{q_p(\theta_1)}{q_p'(\theta_1)} = \frac{M(\theta_1)}{\theta_1}\,\frac{(\theta_1)^2}{\theta_1 M'(\theta_1) - M(\theta_1)} = -\frac{\theta_1 M(\theta_1)}{M(\theta_1) - \theta_1 M'(\theta_1)} = -\frac{\theta_1}{1 - \xi(\theta_1)}. \tag{B.3}$$

18) In this appendix, denote $\theta_1 = \theta_1(\eta)$ and $\theta_2 = \theta_2(\eta)$. On calculating $d\theta_1/d\eta$, express q_p as $q_p(\theta_1)$.

By substituting (B.3) into (B.2), we obtain

$$\frac{d\,\theta_1}{d\,\eta} = -\frac{\theta_1\,d\,\phi_b(\eta)/d\,\eta}{[\,r+s+z+\phi_b(\eta)\,]\,(1-\xi(\theta_1))}.$$

By using this result, the numerator of (B.1) becomes

$$- \frac{\theta_1}{[\,r+s+z+\phi_b(\eta)\,](1-\xi(\theta_1))}\frac{d\,\phi_b(\eta)}{d\,\eta} - \frac{(1-\eta)\,\theta_1\,M(\theta_1)}{(s+z+\phi_b(\eta))[\,r+s+z+\phi_b(\eta)\,]}\frac{d\,\phi_b(\eta)}{d\,\eta} + \frac{\theta_1\,M(\theta_1)}{s+z+\phi_b(\eta)} + \frac{(1-\eta)\,\theta_1\,M(\theta_1)}{(s+z+\phi_b(\eta))^2}\frac{d\,\phi_b(\eta)}{d\,\eta}. \tag{B.4}$$

Comparison of the first term with the third term

We obtain $d\,\phi_b(\eta)/d\,\eta$ as follows:

$$\frac{d\,\phi_b}{d\,\eta} = M(\theta_2)\left[1+\frac{\theta_2\,M'(\theta_2)}{M(\theta_2)}\left(\frac{\eta}{\theta_2}\frac{d\,\theta_2}{d\,\eta}\right)\right] \Rightarrow 0 < \frac{d\,\phi_b(\eta)}{d\,\eta} < M(\theta_2).$$

Then we have

$$\begin{aligned}
&- \frac{\theta_1}{[\,r+s+z+\phi_b(\eta)\,](1-\xi(\theta_1))}\frac{d\,\phi_b(\eta)}{d\,\eta} + \frac{\theta_1\,M(\theta_1)}{s+z+\phi_b(\eta)} \\
&> - \frac{\theta_1\,M(\theta_2)}{[\,r+s+z+\phi_b(\eta)\,](1-\xi(\theta_1))} + \frac{\theta_1\,M(\theta_1)}{s+z+\phi_b(\eta)} \\
&= \frac{\theta_1\,\{[\,r+s+z+\phi_b(\eta)\,](1-\xi(\theta_1))\,M(\theta_1) - (s+z+\phi_b(\eta))M(\theta_2)\}}{(s+z+\phi_b(\eta))[\,r+s+z+\phi_b(\eta)\,](1-\xi(\theta_1))}, \\
&> 0, \quad \text{if} \quad 1 - \frac{(s+z+\phi_b(\eta))M(\theta_2)}{[\,r+s+z+\phi_b(\eta)\,]\,M(\theta_1)} \geq \xi(\theta_1).
\end{aligned}$$

That is, the sum of the first term and the third term in (B.4) becomes positive when the value of the elasticity $\xi(\theta_1)$ is sufficiently small.

Comparison of the second term with the fourth term

Summing up the second term and the fourth term yields

$$\frac{r\,(1-\eta)\,\theta_1\,M(\theta_1)}{(s+z+\phi_b(\eta))^2[\,r+s+z+\phi_b(\eta)\,]}\frac{d\,\phi_b(\eta)}{d\,\eta} > 0.$$

These results indicate that the RHS of (24) is increasing in η when $\xi(\theta_1)$ is sufficiently small.

C. Proof of Proposition 2

The purpose of this section is to show that there exists a unique value of η satisfying (24). This is described by an intersection point of two curves derived from (24), which are functions of η.

By denoting the RHS of (24) as $T(\eta)$, we obtain from definitions of $\phi_p(\eta)$ and $\phi_b(\eta)$

$$T(0) \;=\; \frac{(s+z)\,\theta_1(0)}{s+z+M(\theta_1(0))}, \quad T(1) \;=\; \theta_1(1).$$

To ensure the unique existence of η satisfying (24), the following boundary conditions are necessary:

$$T(0) \;<\; \theta_2(0) \quad \text{and} \quad T(1) \;>\; \theta_2(1).$$

See Figure 1 for the illustration of these conditions.

We first consider the case $\eta = 1$. It follows from (16′) and (17′) that

$$\frac{[\,r+s+z+M(\theta_2)\,]\,c_p}{\lambda\, y - \bar{w}} \;=\; q_p(\theta_1), \quad \frac{[\,r+s+\beta\, M(\theta_2)\,]\,c_b}{(1-\beta)(y-\bar{w})} \;=\; q_b(\theta_2).$$

At $\eta = 1$, $\theta_1 > \theta_2$ holds when λ is high and c_p is small. Since we then have $q_p(\theta_1) < q_b(\theta_2)$, the decreasing property of $q(\cdot)$ results in $\theta_1 > \theta_2$ (q_p and q_b are identical functions in that they have same mathematical properties).

On the other hand, when $\eta = 0$, the boundary condition is represented by

$$\theta_2(0) > T(0) \iff \theta_2(0) > \frac{(s+z)\,\theta_1(0)}{s+z+M(\theta_1(0))}. \tag{C.1}$$

By differentiating the term located at the far right side with respect to z, its numerator is given by

$$\left[\theta_1 + (s+z)\,\frac{\partial\,\theta_1}{\partial\, z}\right](s+z+M(\theta_1)) - (s+z)\,\theta_1\left(1 + M'(\theta_1)\frac{\partial\,\theta_1}{\partial\, z}\right),$$
$$= \frac{\theta_1\, M(\theta_1)\,\{r\,[\,M(\theta_1) - \theta_1\, M'(\theta_1)\,] \,-\, (s+z)^2\}}{(r+s+z)[\,M(\theta_1) - \theta_1\, M'(\theta_1)]}.$$

This value becomes negative when r is sufficiently small, and in that case, $\partial T(0)/\partial z$ is also negative at $\eta = 0$. Therefore, (C.1) is more likely to be satisfied for a higher z.

Summing up the main points in this appendix, we first require that for a large λ, $\theta_1(1) > \theta_2(1)$ holds for a small value of c_p such that

$$[\,r+s+\beta\, M(\theta_2)\,]\,c_b > [\,r+s+z+M(\theta_2)\,]\,c_p \;\Rightarrow\; \frac{[\,r+s+\beta\, M(\theta_2)\,]\,c_b}{r+s+z+M(\theta_2)} > c_p. \tag{C.2}$$

On the other hand, $\theta_2(0) > T(0)$ is obtained at a high z. However, the values of z and c_p have a mutual relationship through the condition (C.2) for $\theta_1(1) > \theta_2(1)$. Thus, the existence of an equilibrium is assured only when z is neither too high nor too low.

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