Properties of a Wage-Posting Equilibrium with Non-Pecuniary Job Characteristics

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Abstract

An extension to the Burdett-Mortensen type wage-posting model is proposed such that the search activity of workers is based not only on wage payments, but also on a non-pecuniary job benefit that is heterogeneous among employers and known only to workers. With this extension, the model yields a unique wage-posting equilibrium, as in the standard model, and derive an equilibrium wage offer distribution which is explicitly characterized by the profit equivalence condition represented by a differential equation. The resulting wage offer distribution, however, has a decreasing density function; this outcome is consistent with some empirical findings, but is never obtained in the standard model with equally productive agents. Importantly, this latter property of our extension does not depend on productivity differentials among employers, but arises from the introduction of the non-pecuniary job benefit alone.

keywords: wage-posting game; equilibrium wage offer distribution; non-pecuniary job characteristics

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1 Introduction

This paper extends the Burdett-Mortensen wage-posting model such that the search activity of workers is based not only on wage payments, but also on a non-pecuniary job benefit that is heterogeneous among jobs and only observable by workers. We show that our extension yields a unique wage-posting equilibrium and a unique wage offer distribution with a decreasing wage density, even if the productivity of each firm is identical.

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Job search theory plays an important role in explaining the wage differentials among workers with identical productivity. [1] In Burdett and Mortensen (1998), wage dispersion is the equilibrium outcome characterized by the firm's optimal decisions of all firms within the model. In other words, wage differentials in their model are represented by a continuous probability distribution that results from the firms' profit-maximizing behavior. (Hereafter, we refer to the model with identical players and wages that only concern workers as the Burdett-Mortensen model or the standard Burdett-Mortensen model.)

Since the publication of Burdett and Mortensen's seminal paper, many researchers have extended their model in efforts to explain various kinds of problems in the labor market. [2] Their framework is particularly helpful in explaining the problem of wage differentials, in that we can analytically derive a continuous wage offer distribution to represent an equilibrium wage dispersion even if all workers are identical and all employers are identical. (Intuitively, wage differentials might seem to depend on a heterogeneous element among agents such as reservation wages of unemployed workers.) In their model, the on-the-job search behavior of employed workers results in the dispersal of their reservation wages. In addition, it is presumed that employers determine their wage level before a match is made, [3] which causes their profits to depend on the expected number of employed workers as well as offered wages. Burdett and Mortensen show that in this context, a firm will be indifferent between a strategy of "pay high wages and hire a lot of workers" and one of "pay less and hire less". The Burdett and Mortensen model produces infinitely many ways for each firm to make the same profit, and the result is wage dispersion among homogenous players.

The Burdett-Mortensen model, however, suffers a serious shortcoming concerning the shape of its wage offer density function. Its density function is shown to be increasing with respect to offers, but some empirical data suggests that the wage offer density function is approximated instead by a single-peaked function with a long rightward tail (see Bowlus, Kiefer and Newmann (1995), Van den Berg and Ridder (1998) and Mortensen (2003)) or a decreasing function (see Bontemps, Robin and van den Berg (2000)). The model therefore needs some adjustment to be consistent with empirical data. According to these previous researches, the introduction of heterogeneity associated with the productivity of firms can provide a more realistic equilibrium wage offer distribution.

1) Mortensen and Pissarides (1999) survey variants of the wage-posting model, and Mortensen (2003) uses this approach to examine the cause of wage dispersion both theoretically and empirically.

2) Rosholm (2000), Galindo-Rueda (2002) and Quercioli (2005) combine the Burdett-Mortensen model and the human capital investment. Tudela (2004) considers the situation in which an employer's wage determination is influenced by the worker's employment history.

3) In contrast, the wage determination in a matching model is based on the ex post division of surplus among a worker and an employer. This is ordinarily characterized by the Nash bargaining solution. See Pissarides (2000).

This approach succeeds in resolving the discrepancy, but does nothing to explain wage dispersion among equally productive players, which may take the form of intra-industry wage differentials. Groshen (1991) suggests that intra-industry establishment wage differentials can explain a large portion of wage variation. According to Lang (1991), productivity differentials of employers are not presumed in formulating a model that intends to account for intra-industry variation in wages. These literatures support the position of assuming identical firms with respect to productivity. Consequently, we introduce to the standard Burdett-Mortensen model a non-wage value of a job, which is presumed to be known only to workers (that is, their private information). By paying attention to the impact of its presence on the equilibrium wage offer distribution, we may then examine the properties of wage differentials when employers are equally productive.

Several researchers have called attention to the importance of non-wage factors to a worker's decision making process. Woodbury (1983), Blau (1991) and Ophem (1991) consider non-wage job components to be those that are controlled by firms, such as fringe benefits and working hours. Blau suggests that it is reasonable to capture workers' search behavior in reservation utility rather than reservation wage, while Hwang, Mortensen and Reed (1998) combine the Burdett-Mortensen sequential search model with the hedonic wage theory to show that salary need not increase as the level of amenity declines. [4)] In their model, the non-pecuniary job benefits (called job amenity) is a control variable for firms, and contracts offered by firms are composed of both wages and amenities. Since the total benefit of the job received by a worker is the weighted sum of these elements, he accepts the offer if it exceeds the reservation his reservation utility level. [5)] On the other hand, Albrecht and Vroman (1992) and Jellal and Zenou (1999) show that workers ought to make decisions about exerting effort, quitting or changing jobs based on a non-pecuniary job benefit that is presumed to be their private information, in addition to an offered wage. These researchers show that incorporating this non-pecuniary characteristic enriches the model. Could it not also have interesting effects on the wage-posting framework ? [6)]

To put it concretely, we assume that a non-pecuniary job benefit (or a non-wage benefit that is not pecuniary) is a worker's private information, and that it may be regarded as a random variable for workers engaging in job search behavior. Once workers find jobs and

4) For example, according to Cahuc and Zylberberg (2004), the hedonic wage theory can explain a wage differential that serves as compensation for a difference in job difficulty. But Hwang, Mortensen and Reed throw this claim into doubt, and Lang and Majumdar (2004), using the non-sequential search model, concur that there need not be a negative relationship between salary and non-pecuniary job benefits.

5) Burdett and Wright (1998) and Masters (1999) focus on the transferability of utility, which is composed of a pecuniary benefit and a non-pecuniary benefit, and specify conditions for a unique market equilibrium. But they assume the wage is exogenous.

6) Albrecht and Vroman (1992) concentrate on showing the existence of dual labor markets in the search framework, and Jellal and Zenou (1999) focus on the efficiency wage model and examine the validity of the Solow condition.

accept offers, they feel satisfaction or dissatisfaction from these jobs and this is reflected in their flow utility. In this context, we can show the existence of a unique wage-posting equilibrium and an equilibrium wage offer distribution. Furthermore, the extended model characterizes a decreasing density function. Importantly, this result holds even if all workers and all employers are equally productive, respectively.

Albrecht and Vroman (1998) also find a continuous wage offer distribution with a decreasing density function that is endogenously determined in the labor market equilibrium. However, the expression characterizing the equilibrium wage offer distribution is too complicated to be analytically tractable. [7] Our model, by contrast, is constructed by extending the Burdett-Mortensen framework, and it follows that we can not only show that a unique equilibrium wage distribution exists with a decreasing density function, but also identify the concrete shape of these functions. Furthermore, in spite of the assumption that firms cannot know the non-pecuniary value of their own jobs, we can show that under certain conditions, firms can at least offer workers their reservation utilities. While these results are similar to those of the standard model, our model arrives at these results by different means. In short, we can demonstrate that the introduction of a non-pecuniary job characteristic to the model serves to describe the labor market more realistically. [8]

The organization of this paper is as follows. In Section 2, we describe some basic presumptions about both workers and employers, and characterize the profit equivalence condition for deriving a wage offer distribution. In Section 3, we solve the differential equation derived in the previous section and specify the concrete shape of the distribution. In Section 4, we describe a wage offer density from the equilibrium offer distribution show that this function can be decreasing in offers under certain conditions. Finally, in Section 5, we offer our conclusions.

2 The Model

We begin with a description of our model. There are many workers and firms in the labor market, and workers engage in job search activity regardless of their employment status. Firms do not search; instead they post wage offers. Let m be a measure of the size of the labor force in the market, defined in proportion to the number of firms. Furthermore, all workers are assumed to be equally productive. All firms also have the same production

7) They develop a model extending the Shapiro-Stiglitz shirking model by introducing heterogeneity with respect to the disutility of effort. This approach combines a moral hazard problem and an adverse selection problem, which greatly complicates their model's structure.

8) Note that we only examine how the presence of this characteristic changes wage dispersion; we do not concern ourselves here with the total utility distribution. Because the available empirical data consists only of offered and earned wages, it is difficult to describe an empirically consistent distribution of total utility.

ability, but they differ with respect to a non-pecuniary component - that is, workers gain utility other than wage earnings from each job. We develop this model under continuous time and focus only on the steady state.

2.1 Worker's Behavior

When engaging in search activity, workers consider a non-wage characteristic of a job in addition to a wage. Let us denote this non-wage characteristic θ, and presume it is distributed across employers. We suppose that this is a random variable with a cumulative distribution function $H(\cdot)$, and that there is a continuous density function denoted by $h(\cdot)$ on the support of it. The support of $H(\cdot)$ is defined to be $[\,\underline{\theta},\, \bar{\theta}\,]$, where $\bar{\theta}$ and $\underline{\theta}$ satisfy the condition $\underline{\theta} < 0 < \bar{\theta}$. Here, the variable θ captures satisfaction or dissatisfaction at a particular job, which is assumed to be unobservable by the employer. In other words, only a worker who receives a job offer can observe the θ corresponding to that job. [9] On the other hand, wage offers made by firms have a distribution function $F(\cdot)$, and firms can not condition their wage offers on their non-pecuniary component since it is known only to workers. Our presumptions of the non-pecuniary aspect of jobs are similar to those of Albrecht and Vroman (1992) in that even after workers succeeds in being matched, they are assumed to engage in the on-the-job search behavior using the same the non-pecuniary characteristics of jobs as they do when they are unemployed. That is, the value of θ does not change with employment status.

Let w denote a wage income. A worker receives flow utility $z \equiv w + \theta$ when employed, and his acceptance decision depends on how the z brought by an offer compares to his reservation utility. Since a firm cannot condition its offer on the non-wage element θ, the probability distribution of z is calculated from $F(\cdot)$ and $H(\cdot)$ using a convolution rule. [10] The density function of z, which we denote by $\phi(\cdot)$, is given by

$$\phi(z) \;=\; \int_{\underline{w}}^{\bar{w}} h(z - w)\, f(w)\, dw,$$

where the support of $F(\cdot)$ is described as $[\,\underline{w},\, \bar{w}\,]$ and that of $\Phi(\cdot)$, which is a c.d.f of z, is given by $[\,\underline{z},\, \bar{z}\,]$. $f(\cdot)$ represents a density function of $F(\cdot)$, so the cumulative distribution function $\Phi(\cdot)$ is the integral of $\phi(\cdot)$ in the interval $[\,\underline{z},\, \tilde{z}\,]$. The upper and lower bounds of z will have the properties that $\bar{z} \leq \bar{w} + \bar{\theta}$ and $\underline{z} \geq \underline{w} + \underline{\theta}$ respectively. Since every firm

9) Hwang, Mortensen and Reed (1998) introduce hedonic wage theory into the Burdett-Mortensen model, and criticizes the traditional statement of that theory. The non-pecuniary benefit of a job is regarded as an amenity, and assumed to be a control variable of firms in their model. In addition, it is observable by all agents.

10) The application of the convolution requires the existence of a density function of $F(\cdot)$. This is not currently unproven, but it is the outcome characterized in the wage-posting equilibrium. In fact, we can derive it in the later section, and temporarily assume its presence.

has to provide a wage offer contained in $[\underline{w},\, \bar{w}\,]$ in equilibrium, $\bar{z}$ ($\underline{z}$) is not always equal to $\bar{w}+\bar{\theta}$ ($\underline{w}+\underline{\theta}$). But for simplicity, let us presume that a worker's best job provides him (her) total utility $\bar{w}+\bar{\theta}$. [11)]

Bellman equations for both unemployed and employed workers can be written as follows. The value of the unemployment state V_0 is

$$r\,V_0 \;=\; b + \;\lambda_0\;\left[\int\; \max\{V_0,\; V_1(\tilde{z})\}\, d\Phi(\tilde{z}) - V_0\right], \tag{2.1}$$

and that of the employment state V_1 is

$$r\,V_1(z) \;=\; z + \;\lambda_1\;\left[\int\; \max\{V_1(z),\; V_1(\tilde{z})\}\, d\Phi(\tilde{z}) - V_1(z)\right] + \delta\,[\,V_0 - V_1(z)\,], \tag{2.2}$$

where b is the unemployment benefit, λ_0 and λ_1 are the arrival rates of jobs for unemployed and employed workers, respectively, and δ is an exogenous separation rate. Note that expectation in (2.1) and (2.2) depends on the distribution of the total utility rather than the distribution of wage offers.

Since we are only extending the Burdett-Mortensen model, a worker's acceptance decision is similar to the one described therein. Our model is also similar to Hwang, Mortensen and Reed (1998); as in theirs, the facts that $V_1(z)$ is increasing in z, that V_0 is independent of a received offer, and that the reservation value R is defined such that $V_1(R) = V_0$, assure the existence of this R:

$$V_1(z) \geq V_0 \qquad \text{as } z \geq R \;\text{ and }\; V_1(z) < V_0 \qquad \text{as } z < R. \tag{2.3}$$

Note that a higher wage offer will not always lead to a match in our model, since workers may consider the firms making this offer to have poorer job conditions as expressed by a low θ (the value of which an employer cannot know, due to our presumption of informational asymmetry).

From (2.1) and (2.2), we derive the following equation characterizing the reservation utility of workers, R:

$$R - b \;=\; [\,k_0 - k_1\,] \int_R^{\bar{z}} \left[\frac{1-\Phi(\tilde{z})}{1 + k_1\,[\,1-\Phi(\tilde{z})\,]}\right] d\tilde{z} \tag{2.4}$$

where we define k_0 as λ_0/δ and k_1 as λ_1/δ. In addition, we assume that r/λ_0 to be 0 for simplicity. The unemployment rate u in the steady state is determined by the point where

11) As we will show later, the assumption about $\bar{z}$ ensures that $\underline{z}$ is equal to R, the worker's reservation utility. Unless a firm provides total utility greater than the reservation level, it cannot hire any workers, obviously nor make any profits. Since an employer knows the values of $\bar{\theta}$ and $\underline{\theta}$, which constitute the support of $H(\cdot)$, it will offer the wage payments that at least compensate workers their reservation utility based on its knowledge of total utility. Furthermore, it is reasonable to presume $\bar{z} = \bar{w}+\bar{\theta}$, because it has a one-to-one relationship with $\underline{z} = R$.

the flow out of the unemployment pool, $\lambda_0 \, [\, 1 - \Phi(R) \,] \, u$ is equal to the inflow, $\delta \, (m - u)$. This is expressed as follows:

$$u = \frac{m}{1 + k_0 \, [\, 1 - \Phi(R)]}. \tag{2.5}$$

We must note that the right-hand side of (2.5) depends on the distribution $\Phi(\cdot)$ rather than the distribution of wage offers.

2.2 Employer Behavior

Probability of Success at a Given Wage Level

Our extended model differs from others in that firms must make wage offers based in part on their non-wage job components, even though these are unknown to them. [12] Therefore, we must define the probability employers face that workers will accept a wage offer; in other words, when firms offer a wage $\hat{w}$, they need to know the probability that the total value of z will exceed the reservation level. For unemployed workers, this is equivalent to the probability that θ is greater than $R - \hat{w}$. This probability, which we denote $\Psi(\hat{w})$, is expressed by

$$\Psi(\hat{w} \,|\, R) \;=\; \int_R^{\bar{z}} h(z - \hat{w}) \, dz \;=\; 1 - H(R - \hat{w}), \quad \text{for} \quad \forall \hat{w} \in [\, \underline{w}, \, \bar{w} \,].$$

Since a firm does not know its own θ, it cannot be certain whether an offer will be accepted. The above expression shows that as firms offer higher wages, the likelihood increases that the total value of the job offers as seen by workers will exceed the reservation value R.

Since a firm's profit depends on the number of new hires obtained by offering a wage $\hat{w}$, we now solve for this number, and characterize the firm's profit maximization problem as in the standard models. According to Burdett and Mortensen (1998) and Hwang, Mortensen and Reed (1998), it is important to define the probability employers face that workers will accept a wage less than or equal to some specified level $\hat{w}$; this decision depends on whether their total utility is greater than their reservation level. This probability, which we denote $G(\cdot)$, is given as

$$G(\hat{w}) \;=\; \int_{\underline{w}}^{\hat{w}} \Psi(w \,|\, R) \, dF(w) \;=\; F(\hat{w}) \;-\; \int_{\underline{w}}^{\hat{w}} H(R - w) \, dF(w).$$

In other words, $G(\cdot)$ is the probability, as seen by firms, that workers will accept an offer which is less than or equal to $\hat{w}$. Similarly, the probability that total utility will exceed a

12) We would expect many wage offer distributions, as in Hwang, Mortensen and Reed (1998), if employers knew the value of their θ.

worker's reservation utility when an employer makes him an offer greater than or equal to some $\hat{w}$ is defined by

$$K(\hat{w}) \;=\; \int_{\underline{w}}^{\bar{w}} \Psi(w \,|\, R)\, dF(w) \;=\; 1 - \int_{\hat{w}}^{\bar{w}} H(R-w)\, dF(w) \;-G(\hat{w}). \tag{2.6}$$

The value of the second term in the right hand side of (2.6) depends on the scale of the wage offer. That is, if every wage in the support of $F(\cdot)$ were greater than $R-\underline{\theta}$, this term could be zero. As we will show later, only the best offer must be equal to $R-\underline{\theta}$, so that term must be strictly positive.

We now ask the question of how each firm will estimate its own non-wage characteristic and its current position. In the standard Burdett-Mortensen model, it is sufficient for employers to take only wage payments into consideration. Because employers do not have access to very much useful information, it is difficult for them to estimate the total utility of the jobs they offer, and therefore for them to guess whether this value is greater than or equal to workers' reservation value R. We assume that a firm regards the turnover behavior of its employees as follows: When an employee receives a wage offer that is not only higher than the present one, but also provides greater total utility than his reservation utility R, he will move to another job. This is almost the same as the setup in the Burdett-Mortensen model, with the exception of the condition with respect to the level of total utility. That is, the wage level is the only information a firm can use to estimate expected worker turnover. In this regard, it is necessary for us to calculate $K(\hat{w})$ as given by (2.6). In the following argument, we adapt this idea to the firm's decision problem.

Let $I(\hat{w})$ be the firm's expectation, from its point of view, of the ratio of employed workers receiving payment less than or equal to $\hat{w}$. The dynamics of $I(\hat{w})$ with inflow to and the outflow from the unemployment pool determine its steady state value:

$$\frac{d\, I(\hat{w}\,;\, t)(m-u(t))}{dt} \;=\; \lambda_0\, G(\hat{w})\, u(t) \;-\; [\,\delta + \lambda_1\, K(\hat{w})\,]\, I(\hat{w}\,;\, t)\,(m-u(t)).$$

The first term on the right-hand side represents the workers who find jobs paying wages less than or equal to $\hat{w}$. The second term is the flow of employees who move to another better jobs or leave due to exogenous reasons. Since the time derivative of $I(\cdot)$ is zero in the steady state, so obtain

$$I(\hat{w}) \;=\; \frac{k_0\, G(\hat{w})\, u}{[\,1 + k_1\, K(\hat{w})\,]\,(m-u)}. \tag{2.7}$$

Our purpose is to derive the expected numbers of new employees by using (2.7). Let us denote this value $L(\hat{w} \,|\, F,\, K,\, I)$. Then, according to Burdett and Mortensen (1998), we may specify the following expression using (2.5), (2.6) and (2.7):

$$\lim_{\varepsilon\to 0} L(\hat{w} \,|\, F,\, K,\, I) \;=\; \lim_{\varepsilon\to 0} \frac{I(w) - I(w-\varepsilon)}{F(w) - F(w-\varepsilon)}\,(m-u) \;\equiv\; L(\hat{w} \,|\, F,\, K,\, \Phi), \tag{2.8}$$

representing the number of workers the employer expects to hire at wage $\hat{w}$. The derivation process for this value is the same as in the standard Burdett-Mortensen models, in which $L(\cdot\,|\,\cdot)$ is shown to be

$$L(\hat{w}\,|\,F,\,K,\,\Phi)\;=\;\frac{k_0\,m\,(1+k_1\,\Omega)}{[\,1+k_1\,K(\hat{w})\,]\,[\,1+k_1\,K(\hat{w}-)\,]\,[\,1+k_0\,(1-\Phi(R))\,]},\tag{2.9}$$

where

$$K(\hat{w}-)\;\equiv\;\Omega-\lim_{\varepsilon\to 0}G(\hat{w}-\varepsilon)\;\text{ and }\;\Omega\;\equiv\;1-\int_{\underline{w}}^{\bar{w}}H(R-w)\,dF(w).$$

From the definitions of $G(\cdot)$ and $K(\cdot)$, we see that $G(\hat{w})$ is increasing in $\hat{w}$ while $K(\hat{w})$ is decreasing in $\hat{w}$. Since lower values of $K(\hat{w})$ yield higher values of (2.9), it follows that $L(\cdot\,|\,F,\,K,\,G)$ is increasing in $\hat{w}$. This means that employers face a trade-off between a strategy of "high wages and high employment" and one of "low wages and low employment," just as in the standard Burdett-Mortensen model.

Equilibrium Wage Offer Distribution

Expression (2.9) allows us to characterize a firm's optimization problem. Only after employees post their wage offers, obviously, can they hire workers who accept those offers. Here, the only input for production is the labor force, and the productivity per worker is assumed to be a constant, y. Since the profit per worker is $y-w$, the total profit of an employer is the product of this per-worker profit and the number of its employees, which is given by (2.9). For a wage offer $\hat{w}$,

$$\pi\;\equiv\;\max_{\hat{w}}\;\pi(\hat{w})\;\equiv\;\max_{\hat{w}}\;L(\hat{w}\,|\,F,\,K,\,\Phi)\,(y-\hat{w}),\tag{2.10}$$

where $y>b$ (to exclude the trivial case). Therefore, we can describe the firm's problem as follows:

$$\begin{aligned}L(\hat{w}\,|\,F,\,K,\,\Phi)\,(y-\hat{w})\;&=\;\pi\;\;\text{for all }\hat{w}\,\in\,[\,\underline{w},\,\bar{w}\,],\\ L(\hat{w}\,|\,F,\,K,\,\Phi)\,(y-\hat{w})\;&\le\;\pi\;\;\text{otherwise.}\end{aligned}$$

These expressions tell us that in market equilibrium, a firm must profit equally from paying any offer contained in the support of $F(\cdot)$. The support does not contain offers which would result in lower profits. Hence, the profit-maximizing behavior of firms is to offer a wage contained in the support of $F(\cdot)$.

We now turn our attention to the shape of the distribution $F(\cdot)$ and of its support. Before determining the existence of an equilibrium wage offer distribution, we confirm that this distribution has no probability mass in the first place. We may confirm this by supposing that $\tilde{w} \in [\,\underline{w},\, \bar{w}\,]$ is a mass point of $F(\cdot)$, and then showing a contradiction. It follows from the supposition that we can decompose $F(\tilde{w})$ to $F(\tilde{w}) = F(\tilde{w}-) + \bar{\mu}$. The second term of this expression stands for the probability mass. It can be shown that if a firm slightly raises its offer from $\tilde{w}$, it can achieve strictly greater profits. This is in contradiction to our premise that every offer contained in $[\,\underline{w},\, \bar{w}\,]$ must be profit-maximizing; therefore, the support must contain $\tilde{w}$. This is the proof described in Burdett and Mortensen (1998) and summarized by Quercioli (2005). Referring to these existing literatures, we investigate the sign of the following expression:

$$\lim_{\varepsilon \to 0} \{L(\tilde{w} + \varepsilon \,|\, F,\, K,\, \Phi)\,[\,y - (\tilde{w} + \varepsilon)\,] \,-\, L(\tilde{w} \,|\, F,\, K,\, \Phi)\,(y - \tilde{w})\}\,. \tag{2.11}$$

Simple calculations indicate that (2.11) is strictly positive. [13] The arguments above show that the additional profit realized by paying more than $\tilde{w}$ is strictly positive, since the profit per worker is invariable as ε tends to 0 and employment is strictly increasing. Since the choice of a mass point is arbitrary, the distribution $F(\cdot)$ must be continuous on its support. Furthermore, as pointed out in Burdett and Mortensen (1998) and Tudela (2004), we can easily prove that the support of $F(\cdot)$ is a connected set. These arguments suggest the following proposition:

Proposition 1
The distribution of wage offers is everywhere continuous on its support, and in addition, this support is a connected set.

We have illustrated some properties of the wage offer distribution, but we have not yet investigated the support of this distribution. Since the total value of a job in this model depends on a component which is unknown to employers, the lower bound of the support is not simply equal to the reservation wage, as it is in the standard models. We will characterize the support in a later section.

Our next task is to find out an expression to characterize the equilibrium wage offer distribution. The expected profit π of a firm paying $\bar{w}$ is

$$\pi(\bar{w}) \;=\; \left[\, \frac{k_0\, m\,(1 + k_1\,\Omega)}{1 + k_0\,(1 - \Phi(R))} \,\right]\,(y - \bar{w})\,,$$

where we have already substituted $F(\bar{w}) = 1$. Profit-maximization behavior requires that the firm's profit should be equivalent across all wages in the support of $F(\cdot)$; $\pi(\bar{w}) = \pi(\hat{w})$ for every $\hat{w}$ contained in $[\,\underline{w},\, \bar{w}\,]$. This means

$$\frac{y - \bar{w}}{y - \hat{w}} \;=\; \frac{1}{[\,1 + k_1\,K(\hat{w})\,]^2}, \qquad \forall\, \hat{w} \in \,[\,\underline{w},\, \bar{w}\,]. \tag{2.12}$$

13) See, for example, Burdett and Mortensen (1998), Tudela (2004) and Quercioli (2005) for more precise proofs.

Note that $K(\cdot)$ can be written as follows:

$$K(\hat{w}) \;=\; 1 - F(\hat{w}) - \int_{\hat{w}}^{\bar{w}} H(R-w)\, dF(w),$$
$$\;=\; 1 - F(\hat{w}) - H(R-\bar{w}) + H(R-\hat{w})\, F(\hat{w}) - \int_{\hat{w}}^{\bar{w}} h(R-w)\, F(w)\, dw. \quad (2.13)$$

We derive the second equality by developing the integral in the first. (2.12) and (2.13) indicate that the derivation of an explicit expression for the wage offer distribution from the profit equivalence condition is more difficult in our model than in the standard Burdett-Mortensen model. But if we suppose $H(\cdot)$ to be uniform $-$ that is, $H(\theta)$ is given by $(\theta - \underline{\theta})/(\bar{\theta} - \underline{\theta})$ and $h(\theta)$ is described by $1/(\bar{\theta} - \underline{\theta})$. We can then find an explicit expression. From (2.13) - this additional constraint yields

$$1 + k_1\, K(\hat{w}) \;=\; A - \frac{k_1(\hat{w} - w')}{\bar{\theta} - \underline{\theta}}\, F(\hat{w}) - \frac{k_1}{\bar{\theta} - \underline{\theta}} \int_{\hat{w}}^{\bar{w}} F(w)\, dw, \quad (2.14)$$

where A and w' are defined by

$$A \equiv\; 1 + \frac{k_1\, (\bar{w} - w')}{\bar{\theta} - \underline{\theta}}, \;\; w' \equiv R - \bar{\theta}.$$

By applying (2.14), the profit equivalence condition (2.12) can be rewritten as follows:

$$A + \frac{k_1\, (\hat{w} - w')}{\bar{\theta} - \underline{\theta}}\, \frac{d\tilde{F}(\hat{w})}{d\hat{w}} - \frac{k_1}{\bar{\theta} - \underline{\theta}}\, \tilde{F}(\hat{w}) \;=\; \sqrt{\frac{y - \hat{w}}{y - \bar{w}}}. \quad (2.15)$$

Note that $\tilde{F}(\hat{w})$ is defined as $\int_{\hat{w}}^{\bar{w}} F(w)\, dw$. Although the equation (2.15) is a key condition for characterizing the equilibrium offer distribution, it is a differential equation about $\tilde{F}(\cdot)$ and $\hat{w}$. Therefore, we have to solve this differential equation and obtain the explicit form of $F(\cdot)$. This is the object of the next section.

3 The Equilibrium Wage Offer Distribution

3.1 A Solution to Differential Equation (2.15)

To learn more about the properties of this equation, we can rearrange it as follows:

$$\left(A - \frac{k_1}{\bar{\theta} - \underline{\theta}}\, \tilde{F}(\hat{w}) - \sqrt{\frac{y - \hat{w}}{y - \bar{w}}} \right) d\,\hat{w} \;+\; \frac{k_1(\hat{w} - w')}{\bar{\theta} - \underline{\theta}}\, d\,\tilde{F}(\hat{w}) \;=\; 0. \quad (3.1)$$

In solving a differential equation like (3.1), it is crucial to know whether or not it is exact. If it is not exact, we can find the integrating factor and make it exact by using this factor. In fact, (3.1) is not an exact equation. We know this because of the following basic fact with respect to the exact equations:

Fact 1

Let $P(t,\, x)$ and $Q(t,\, x)$ be twice continuously differentiable functions in $\mathbb{R}^2$. Then the equation $P(t,\, x)\, dt + Q(t,\, x)\, dx = 0$ is exact if and only if the following condition is satisfied:

$$\frac{\partial P(t,\, x)}{\partial x} = \frac{\partial Q(t,\, x)}{\partial t}. \tag{3.2}$$

Furthermore, if this condition is satisfied, then a solution $\phi(t,\, x) = c$ satisfying the above equation is given as below: [14)]

$$\phi(t,\, x) = \int_{t_0}^{t} P(t,\, x)\, dt + \int_{x_0}^{x} Q(t_0,\, x)\, dx, \tag{3.3}$$

where t_0, x_0 and c are arbitrarily constant.

If we regard t as $\hat{w}$ and x as $\tilde{F}$, and define functions P and Q as

$$P(\hat{w},\, \tilde{F}) \equiv A - \frac{k_1}{\bar{\theta} - \underline{\theta}}\, \tilde{F}(\hat{w}) - \sqrt{\frac{y - \hat{w}}{y - \bar{w}}} \quad \text{and} \quad Q(\hat{w},\, \tilde{F}) \equiv \frac{k_1(\hat{w} - w')}{\bar{\theta} - \underline{\theta}}.$$

We see that the partial derivative of P with respect to $\tilde{F}$ is not equal to that of Q with respect to $\hat{w}$. To solve equation (3.1), then, we must find an integrating factor. The equation

$$\frac{\partial P/\partial \tilde{F} - \partial Q/\partial \hat{w}}{Q} = \frac{2}{w' - \hat{w}},$$

tells us that an integrating factor of (3.1) will depend on only $\hat{w}$. This factor, which we denote $\mu(\hat{w})$, can be expressed as

$$\mu(\hat{w}) = \exp\left(- \int_{\underline{w}}^{\hat{w}} \frac{2}{\hat{w} - w'}\, dw \right) = \left(\frac{w' - \underline{w}}{w' - \hat{w}} \right)^2. \tag{3.4}$$

If we now multiply P and Q by $\mu(\hat{w})$ from (3.4) and calculate their partial derivatives with respect to $\tilde{F}$ and $\hat{w}$ respectively, then these two derivatives match, and the equation (3.1) can be written as an exact equation. We can apply Fact 1 to this renewed (3.1), yielding

$$\begin{aligned} \phi(\hat{w},\, \tilde{F}) &= \int_{\underline{w}}^{\hat{w}} \left(\frac{w' - \underline{w}}{w' - w} \right)^2 \left(A - \frac{k_1}{\bar{\theta} - \underline{\theta}}\, \tilde{F}(w) - \sqrt{\frac{y - w}{y - \bar{w}}} \right) dw \\ &\quad - \frac{k_1\, (w' - \underline{w})^2}{\bar{\theta} - \underline{\theta}} \int_{\underline{w}}^{\hat{w}} \frac{1}{w' - w}\, \frac{d\, \tilde{F}(w)}{d\, w}\, dw, \\ &= c. \end{aligned}$$

14) Suppose that a differential equation is given as $P(t,\, x)\, dt + Q(t,\, x)\, dx = 0$. Then, if there exists a twice continuously differentiable function $\phi(t, x)$ such that

$$P = \frac{\partial \phi}{\partial t}, \;\; Q = \frac{\partial \phi}{\partial x},$$

then $\phi(t,\, x) = c$ is known to be the solution of the equation. See Braun (1975).

We rearrange the above integral to express $\phi(\hat{w},\, \tilde{F})$ as follows:

$$\begin{aligned}\phi(\hat{w},\, \tilde{F}) \;&=\; \frac{A\,(w'-\underline{w})^2}{w'-\hat{w}} - A\,(w'-\underline{w}) - \frac{k_1\,(w'-\underline{w})^2}{(\bar{\theta}-\underline{\theta})(w'-\hat{w})}\tilde{F}(\hat{w}) \\ &\quad + \frac{k_1\,(w'-\underline{w})}{\bar{\theta}-\underline{\theta}}\,\tilde{F}(\underline{w}) - \int_{\underline{w}}^{\hat{w}} \left(\frac{w'-\underline{w}}{w'-w}\right)^2 \sqrt{\frac{y-w}{y-\bar{w}}}dw, \\ &=\; c. \end{aligned} \tag{3.5}$$

We can easily show that ϕ is a solution to the equation (3.1); the derivative of ϕ with respect to $\tilde{F}$ is $\mu(\hat{w}) \times Q$, and its derivative with respect to $\hat{w}$ is $\mu(\hat{w}) \times P$. See footnote 14).

Furthermore, (3.5) allows us to solve for $\tilde{F}(\hat{w})$, giving us

$$\begin{aligned}\tilde{F}(\hat{w}) \;&=\; \frac{A\,(\bar{\theta}-\underline{\theta})(\hat{w}-\underline{w})}{k_1\,(w'-\underline{w})} + \left(\frac{w'-\hat{w}}{w'-\underline{w}}\right)\tilde{F}(\underline{w}) - \frac{\bar{\theta}-\underline{\theta}}{k_1}\int_{\underline{w}}^{\hat{w}} \frac{w'-\hat{w}}{(w'-w)^2}\sqrt{\frac{y-w}{y-\bar{w}}}\,dw \\ &\quad - \frac{(\bar{\theta}-\underline{\theta})(w'-\hat{w})}{k_1\,(w'-\underline{w})^2}\,c. \end{aligned} \tag{3.6}$$

The right-hand side of (3.6) is a function of $\hat{w}$, and we can obtain an equilibrium wage offer distribution differentiating it with respect to $\hat{w}$. Before doing so, it will be useful to specify the values of c and $\tilde{F}(\underline{w})$. Substituting $\underline{w}$ and $\bar{w}$ into (3.6) yields $c = 0$ and

$$\tilde{F}(\underline{w}) \;=\; \frac{A\,(\bar{\theta}-\underline{\theta})(\bar{w}-\underline{w})}{k_1\,(\bar{w}-w')} + \frac{(\bar{\theta}-\underline{\theta})(w'-\underline{w})}{k_1}\int_{\underline{w}}^{\bar{w}} \frac{1}{(w'-w)^2}\sqrt{\frac{y-w}{y-\bar{w}}}\,dw.$$

From these expressions, we find the derivative of $\tilde{F}(\hat{w})$ with respect to $\hat{w}$:

$$\begin{aligned}\frac{d\,\tilde{F}(\hat{w})}{d\,\hat{w}} \;&=\; -\,F(\hat{w}), \\ &=\; -\,\frac{A\,(\bar{\theta}-\underline{\theta})}{k_1\,(\bar{w}-w')} - \frac{\bar{\theta}-\underline{\theta}}{k_1\,(w'-\hat{w})}\sqrt{\frac{y-\hat{w}}{y-\bar{w}}} - \frac{\bar{\theta}-\underline{\theta}}{k_1}\int_{\hat{w}}^{\bar{w}} \frac{1}{(w'-w)^2}\sqrt{\frac{y-w}{y-\bar{w}}}\,dw, \end{aligned}$$

Applying the definition of A to the above expression yields

$$F(\hat{w}) = 1 + \frac{\bar{\theta}-\underline{\theta}}{k_1\,(\bar{w}-w')} - \frac{\bar{\theta}-\underline{\theta}}{k_1\,(\hat{w}-w')}\sqrt{\frac{y-\hat{w}}{y-\bar{w}}} + \frac{\bar{\theta}-\underline{\theta}}{k_1}\int_{\hat{w}}^{\bar{w}} \frac{1}{(w'-w)^2}\sqrt{\frac{y-w}{y-\bar{w}}}\,dw, \tag{3.7}$$

which is the equilibrium wage offer distribution in this model.

Proposition 2

There is a unique solution to the differential equation (3.1) *which is derived from the profit equivalence condition. This solution characterizes a unique equilibrium wage offer distribution, described by* (3.7).

3.2 The Support of $F(\cdot)$

While it is not difficult to derive an equilibrium wage offer distribution from the profit equivalence condition in the standard Burdett-Mortensen model, we must follow the above procedure to find the concrete shape of this distribution. In our extended model, firms cannot observe the non-pecuniary characteristic workers receive at each job, and must therefore make their hiring decisions based on imperfect information. Even if a firm makes a wage offer greater than the reservation value, a low value for the job's non-wage factor might negate the utility from the wage, making the offer less attractive to workers. In order to specify the range in which total utility will vary, we need to find the support of $F(\cdot)$. Of course, this support is a component of the equilibrium.

We have characterized the shape of the equilibrium wage offer distribution. Now we find the shape of its support $[\,\underline{w},\ \bar{w}\,]$. Substituting $\hat{w} = \underline{w}$ into (3.7) and using the fact that the distribution $F(\underline{w}) = 0$ yields yield

$$1 + \frac{\bar{\theta} - \underline{\theta}}{k_1\,(\bar{w} - w')} - \frac{\bar{\theta} - \underline{\theta}}{k_1\,(\underline{w} - w')}\sqrt{\frac{y - \underline{w}}{y - \bar{w}}} + \frac{\bar{\theta} - \underline{\theta}}{k_1}\int_{\underline{w}}^{\bar{w}} \frac{1}{(w' - w)^2}\sqrt{\frac{y - w}{y - \bar{w}}}\,dw = 0. \quad (3.8)$$

We can see that $F(\bar{w})$ is always one by substituting $\bar{w}$ into (3.7). Before showing the existence of a unique $\underline{w}$, we identify the value of $\bar{w}$ with the following logic.

Remember that if a firm cannot realize any additional profit by raising its wage payments, it is appropriate to regard its current wage level as the upper limit of the support. Let $R - \underline{\theta}$ be a candidate for the limit. That is, a firm pays at most the sum of the reservation utility and the minimum level of a non-pecuniary job component. The employment level L when a firm pays some $\hat{w}$, which depends on $\hat{w}$ only through $K(\hat{w})$ as described in (2.9), is inversely related to $K(\hat{w})$. In other words, if we can show that $K(R - \underline{\theta})$ is less than or equal to $K(\tilde{w})$ for every $\tilde{w}$ such that $R - \underline{\theta} < \tilde{w}$, the offer $R - \underline{w}$ will define the upper limit of the support. When we insert $R - \underline{\theta}$ for $\bar{w}$, $K(\bar{w}) = 0$ from (2.13). Similarly, calculating $K(\tilde{w})$ evaluating at $\tilde{w}$ as defined above yields

$$K(\tilde{w}) \;=\; 1 - F(\tilde{w}) - H(R - \bar{w}) + H(R - \tilde{w})\,F(\tilde{w}) + \int_{\bar{w}}^{\tilde{w}} h(R - w)\,F(w)\,dw \;=\; 0,$$

satisfying the following conditions: (i) if $\bar{w} < \tilde{w}$, then $F(\tilde{w})$ must be one; (ii) if $\bar{w} = R - \underline{\theta}$, then $H(R - \bar{w})$ must be zero; (iii) if $\bar{w} < \tilde{w}$, then $H(R - \tilde{w})$ must be zero, since $H(R - \tilde{w}) \leq H(\underline{\theta}) = 0$, and $H(\cdot)$ must be nonnegative by definition; and (iv) if $R - \tilde{w} < R - \bar{w}\ (= \underline{\theta}\,)$, then $h(R - w)$ must be zero for $w \in (\bar{w},\ \tilde{w}\,]$ because $H(\cdot)$ is uniform. In short, raising the offer above $\bar{w} = R - \underline{\theta}$ has no effect on a firm's employment level, but reduces the profit per worker and the expected profits of the firm as a whole. Thus, $R - \underline{\theta}$ is the upper limit of the support, and any offer in the interval $[\,\underline{w},\ R - \underline{\theta}\,]$ provides the same profit to the firm.

We can specify the value of $\bar{w}$, but it is more difficult to identify the value of $\underline{w}$ from (3.8). However we can show that there is a unique $\underline{w}$ satisfying (3.8) under $\bar{w} = R - \underline{\theta}$. Since a simple calculation indicates that the derivative of (3.8) with respect to $\underline{w}$ is negative, the existence of $\underline{w}$ is assured. Therefore $\bar{w} = R - \underline{\theta}$ and equation (3.8) together completely characterize the support of the equilibrium wage offer distribution.

Proposition 3
The support of the equilibrium wage offer distribution is $[\underline{w}, \bar{w}]$*, where* $\underline{w}$ *uniquely satisfies equation* (3.8) *and* $\bar{w} = R - \underline{\theta}$.

We must confirm that a reservation utility R exists to satisfy equation (2.4). Suppose that both unemployed and employed workers receive job offers at the same rate in the search process: $k_0 = k_1 \equiv k$. Then, by equation (2.4), $R = b$. We have now uniquely characterized the wage-posting equilibrium in our extended model. Our assumption about the arrival rate of job offers simplifies the determination of the reservation utility, allowing us in the next section to concentrate on the properties of the wage offer density function. [15)]

3.3 The Total Utility Distribution

Though we have characterized the unique wage posting equilibrium, we do not attempt to shed light on the distribution $\Phi(\cdot)$ of total utility z. Remember that our primary purpose here is to examine how the introduction of the non-pecuniary job characteristic affects equilibrium wage dispersion. Nevertheless, there is some information about z we can glean from our work thus far: (1) The distribution $\Phi(\cdot)$ of total utility is uniform because of the definition of the distribution and the assumption that $H(\cdot)$ is uniform. (2) The minimum level of total utility in the market is just equal to the reservation level R, even though firms set wages with no knowledge of their non-wage job characteristics.

This first property results from our assumption that the non-pecuniary component is uniformly distributed among employers; it is difficult to confirm whether this is plausible, since this component is practically unmeasurable in real markets, and in any case is highly subjective. On the other hand, the second property results from the following simple calculation:

$$\Phi(\bar{z}) = 1 = \frac{\bar{z} - \underline{z}}{\bar{\theta} - \underline{\theta}} \quad \Rightarrow \quad \underline{z} = \bar{z} - (\bar{\theta} - \underline{\theta}) \ = \ \bar{w} + \bar{\theta} - (\bar{\theta} - \underline{\theta}) \ = \ R - \underline{\theta} + \underline{\theta} = R.$$

Note our presumption that the best job provides workers with utility $\bar{w} + \bar{\theta}$, and that $\bar{w}$ is represented by $R - \underline{\theta}$, as we have been shown.

15) Our assumption that unemployed workers and employed workers receive job offers at the same arrival rate is supported by Van den Berg and Ridder (1998). Other theoretical researches such as Tudela (2004) and Quercioli (2005) also assume this condition for simplicity.

Hwang, Mortensen and Reed (1998) indicate that the minimum utility level is just equal to the reservation utility of workers when firms are able to control their non-wage benefits to hire new employees. This is not too surprising, as a similar result is obtained in the Burdett-Mortensen model. However, our model suggests that under certain circumstances, firms will give workers at least the reservation utility even when firms neither know nor control their non-pecuniary job characteristics.

4 The Wage Offer Density Function

In this section, we derive a wage offer density function and examine its properties. As we have noted, the increasing wage offer density predicted by the standard Burdett-Mortensen model contradicts some empirical data. Several studies have found that the actual data are more closely approximated by a data wage offer density function that is decreasing or single-peaked with a long rightward tail (see Bowlus, Kiefer and Neumann (1995), Van den Berg and Ridder (1998), Bontemps, Robin and van den Berg (2000) and Mortensen (2003)). Standard Burdett-Mortensen models have been criticized for this apparent flaw, even though researchers have found them extremely useful in making various contributions to the problem of wage dispersion. By introducing differences among firms with respect to productivity, the above researchers have shown that it is possible to obtain a more realistically shaped wage offer density function.

Nevertheless, little is known about situations in which the wage offer density function is decreasing or single-peaked when employers have presumed to have equal productivities. Although it is difficult to find the conditions producing a single-peaked wage offer density function, we can establish the conditions necessary to obtain a decreasing one. Albrecht and Vroman (1998) extends the Shapiro-Stiglitz shirking model by incorporating worker heterogeneity as his private information. They show that under the moral hazard problem and the adverse selection problem, their extension produces a continuous wage offer distribution whose density function is decreasing. Unfortunately, their results are obtained by computational analysis and therefore cannot help us draw firm conclusions regarding the concrete shape of the distribution. By starting from the model Burdett and Mortensen model, however, we can calculate its shape directly.

We first differentiate (3.7) with respect to $\hat{w}$. Then it brings us

$$\frac{d\,F(\hat{w})}{d\,\hat{w}} = \frac{\bar{\theta} - \underline{\theta}}{2\,k\,(\hat{w} - w')\sqrt{(y - \bar{w})(y - \hat{w})}} \equiv f(\hat{w}), \quad \text{for } \forall\,\hat{w} \in [\,\underline{w},\, \bar{w}\,]\,. \tag{4.1}$$

Note that the function given by (4.1) is guaranteed to be well-defined if w' is strictly less than $\underline{w}$; as long as $w' < \underline{w}$, (4.1) is strictly positive for all $\hat{w}$ within $[\,\underline{w},\, \bar{w}\,]$.

Our next task is to determine the conditions under which (4.1) is decreasing in $\hat{w}$. Differentiating $f(\hat{w})$ yields

$$\frac{d\,f(\hat{w})}{d\,\hat{w}} = \frac{\bar{\theta}-\underline{\theta}}{k\,(\hat{w}-w')\,\sqrt{(y-\bar{w})(y-\hat{w})}}\left[\frac{1}{2\,(y-\hat{w})}-\frac{1}{\hat{w}-w'}\right], \quad \text{for } \forall\,\hat{w}\in[\,\underline{w},\,\bar{w}\,]\,. \tag{4.2}$$

By rewriting the right hand side of (4.2) as

$$\frac{\bar{\theta}-\underline{\theta}}{k\,(\hat{w}-w')\,\sqrt{(y-\bar{w})(y-\hat{w})}}\left[\frac{3\,\hat{w}-2y-w'}{2\,(y-\hat{w})(\hat{w}-w')}\right].$$

We see that the slope of $f(\hat{w})$ depends on the sign of the bracketed expression. Finally, to ensure that $f(\hat{w})$ has a negative slope, the following inequality is necessary and sufficient:

$$3\,\bar{w} < 2\,y + w' \quad \text{or} \quad \frac{\bar{\theta}-\underline{\theta}}{2} < y-(b-\underline{\theta}). \tag{4.3}$$

In this inequality, $\bar{w}$ has been defined as $\bar{w} = b-\underline{\theta}$, and w' as $w' = b-\bar{\theta}$. We have also employed the fact that the reservation utility is equal to the unemployment benefit b under $k_0 = k_1$.

Note that we have arrived at inequality (4.3), one might reasonably be induced to ask: what, exactly, is thus signified ? The condition expressed therein implies that the slope of our model's wage offer density function depends on parameters such as y, $\bar{\theta}$, $\underline{\theta}$ and b. Note that the offer density $f(\cdot)$ is likely to be decreasing as (i) as $\bar{\theta}$ decreases given a fixed $\underline{\theta}$, (ii) as $\underline{\theta}$ increases under fixed $\bar{\theta}$, and (iii) as b decreases. Decreasing $\bar{\theta}$ with a fixed $\underline{\theta}$ implies fewer jobs that workers would prefer to their current jobs. Worker turnover would therefore seem to decrease in tandem with $\bar{\theta}$, allowing employers to pay lower wages. We can use analogous logic to show that increasing $\underline{\theta}$ under a fixed $\bar{\theta}$ implies that employers will be forced to pay lower wages. Finally, the condition (4.3) implies that the wage offer density in our model is likely to increase or decrease along with the reservation utility of workers.

The reservation wage of employed workers in the standard Burdett-Mortensen model is a simple, non-compound value that differs from the reservation wage of unemployed workers and that is known to employers. Since a higher wage attracts more potential workers and lowers turnover among existing employees, it is reasonable to expect firms to offer relatively high wages voluntarily (resulting in an increasing wage offer density function). On the other hand, in our model, a lower reservation utility reduces the probability that employees can obtain more favorable offers by engaging in on-the-job search for any current environment z. [16] Since this probability is given by

$$1-\Phi(z) = 1-\frac{z-\underline{z}}{\bar{z}-\underline{z}} = \frac{\bar{z}-z}{\bar{z}-\underline{z}} = \frac{b+\bar{\theta}-\underline{\theta}-z}{\bar{\theta}-\underline{\theta}},$$

16) Firms can expect this even when they do not know the values of their non-wage job components.

it decreases as b tends to zero. Remember that unemployed workers' reservation utility R is their cutoff level regardless of whether they accept any offers. Together, these facts suggest that if the reservation utility decreases enough, employers will choose to offer lower wages. This incentive is represented by the decreasing slope of our wage offer density function.

5 Conclusions

In this paper, we have extended the Burdett-Mortensen wage-posting framework by introducing a non-pecuniary component to jobs that is known to workers but not to firms. We have shown that the density function of wage offers in our model is decreasing with respect to offers, even if all firms are equally productive. Our extended model thus stands in contrast to the standard Burdett-Mortensen model, which results in an increasing wage offer density if all employers are identical. Other researchers have arrived at a decreasing wage offer density function by extending the standard model, as have we, but these alternative models presume the productivity of firms to be heterogeneous.

Because firms in our model cannot observe the non-wage aspect of a job from a worker's point of view, they cannot predict how much total utility workers will derive from matches. Accordingly, we need to solve a differential equation derived from the profit equivalence condition in order to characterize an equilibrium wage offer distribution. Because empirical evidence speaks in support of our model's decreasing wage offer density function, non-pecuniary factors do indeed seem to have major effects on employers' wage offers.

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